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2012-2013

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Introduction

The purpose of this guidance document is support educators during the transition period to full implementation of the Next Generation Mathematics Content Standards and Objectives.

The Next Generation Content Standards and Objectives for Mathematics in West Virginia Schools are aligned to the Common Core State Standards for Mathematics, the culmination of an extended, broad-based effort to fulfill the charge issued by the states to create the next generation of K-12 standards in order to help ensure that all students are college and career ready no later than the end of high school. The Common Core State Standards for Mathematics, the product of work led by the Council of Chief State School Officers (CCSSO) and the National Governors Association (NGA), builds on the foundation laid by the states in their decades-long work on crafting high-quality education standards. In May 2010, the West Virginia Board of Education adopted the Common Core State Standards for Mathematics; shortly thereafter, 85 classroom teachers and representatives of Higher Education faculty began a deep study of this work and placed the content of these Standards into the West Virginia Framework. This group of West Virginia educators found the standards to be research- and evidence-based, aligned with college and work expectations, rigorous, and internationally benchmarked. A particular standard was included in the document only when the best available evidence indicated that its mastery was essential for college- and career-readiness in a globally competitive society.

For over a decade, research studies of mathematics education in high-performing countries have pointed to the conclusion that the mathematics curriculum in the United States must become substantially more focused and coherent in order to improve mathematics achievement in this country. To deliver on the promise of common standards, the standards must address the problem of a curriculum that is “a mile wide and an inch deep.” These Standards are a substantial answer to that challenge. It is important to recognize that “fewer standards” are no substitute for focused standards. Achieving “fewer standards” would be easy to do by resorting to broad, general statements. Instead, these Standards aim for clarity and specificity. Assessing the coherence of a set of standards is more difficult than assessing their focus. William Schmidt and Richard Houang (2002) have said that content standards and curricula are coherent if they are articulated over time as a sequence of topics and performances that are logical and reflect, where appropriate, the sequential or hierarchical nature of the disciplinary content from which the subject matter derives. That is, what and how students are taught should reflect not only the topics that fall within a certain academic discipline, but also the key ideas that determine how knowledge is organized and generated within that discipline. This implies that to be coherent, a set of content standards must evolve from particulars (e.g., the meaning and operations of whole numbers, including simple math facts and routine computational procedures associated with whole numbers and fractions) to deeper structures inherent in the discipline. These deeper structures then serve as a means for connecting the particulars (such as an understanding of the rational number system and its properties). These Standards endeavor to follow such a design, not only by stressing conceptual understanding of key ideas, but also by continually returning to organizing principles such as place value or the properties of operations to structure those ideas.

The sequence of topics and performances that is outlined in a body of mathematics standards must also respect what is known about how students learn. In recognition of this, the development of these Standards began with research-based learning progressions detailing what is known today about how students’ mathematical knowledge, skill, and understanding develop over time. In the early grades there is greater focus and coherence.
West Virginia Next Generation Secondary Mathematics Content Standards and Objectives are organized by student learning progressions. The Standards for Mathematical Practice apply throughout each course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful and logical subject that makes use of their ability to make sense of problem situations.

**Grade 8**

The Next Generation Grade 8 standards are of significantly higher rigor than the former Grade 8 standards. The Next Generation Grade 8 standards address the foundations of Algebra by including content that was previously part of the Algebra I course standards, such as more in-depth study of linear relationships and equations, a more formal treatment of functions, and the exploration of irrational numbers. These standards also include geometry standards that relate graphing to algebra in a way that was not explored in previous standards. In addition, the statistics presented in the Grade 8 Next Generation standards are more sophisticated than those previously included in middle school and connect linear relations with the representation of bivariate data. The new grade 8 standards address more algebra topics than our previous grade 8 standards.

**High School Math 9**

The High School Math 9 course builds on the Next Generation Grade 8 standards and is correspondingly more advanced than our previous Algebra I course. Because many of the topics previously included in the Algebra I course are in the Next Generation Grade 8 standards, the High School Math 9 course starts with more advanced topics and includes more in depth work with linear functions, exponential functions and relationships, transformations and connecting algebra and geometry through coordinates. It also goes beyond the previous high school standards in statistics.

**High School Math 10**

The High School Math 10 course builds on the Math 9 course. The focus is on quadratic expressions, equations, and functions; comparing their characteristics and behavior to those of linear and exponential relationships from Mathematics 9 as organized into six critical areas, or units. The link between probability and data is explored through conditional probability and counting methods, including their use in making and evaluating decisions. The study of similarity leads to an understanding of right triangle trigonometry and connects to quadratics through Pythagorean relationships. Circles and their quadratic algebraic representations, round out the course.
**High School Math 11**

The High School Math 11 course builds on the Math 10 course and offers a more personalized learning plan aligned to students’ career aspirations – Math 11 LA, Math 11 STEM, or Math 11 TR. It is in Math 11 that students pull together and apply the accumulation of learning that they have from their previous courses, with content grouped into four critical areas, organized into units. They apply methods from probability and statistics to draw inferences and conclusions from data. Students expand their repertoire of functions to include polynomial, rational and radical functions. They expand their study of right triangle trigonometry to include general triangles. Finally, students bring together all of their experience with functions and geometry to create models and solve contextual problems.

**The High School Math 12 courses builds on the Math 11 STEM.** The fundamental purpose of Math 12 is to generalize and abstract learning accumulated through previous courses and to provide the final springboard to calculus. Students take an extensive look at the relationships among complex numbers, vectors, and matrices. They build on their understanding of functions, analyze rational functions using an intuitive approach to limits and synthesize functions by considering compositions and inverses. Students expand their work with trigonometric functions and their inverses and complete the study of the conic sections begun in Mathematics 10. They enhance their understanding of probability by considering probability distributions. Previous experiences with series are augmented.

*Math 11 Technical Readiness & Math 12 Technical Readiness are course options (for juniors and seniors) built from the mathematics content of Math 11 through integration of career clusters. These courses integrate academics with hands-on career content. The collaborative teaching model is recommended based at our CTE centers. The involvement of a highly qualified Mathematics teacher and certified CTE teachers will ensure a rich, authentic and respectful environment for delivery of the academics in “real world” scenarios.

**Note:** Additional course options are not limited to AP Calculus, AP Statistics, AP Computer Science, Advanced Mathematical Modeling, STEM Readiness Mathematics, Transition Math for Seniors and Math 12.
**Fourth Course Descriptions**  
(Not Limited to These Courses)

**High School Math 12**  
The fundamental purpose of Mathematics 12 is to generalize and abstract learning accumulated through previous courses and to provide the final springboard to calculus. Students take an extensive look at the relationships among complex numbers, vectors, and matrices. They build on their understanding of functions, analyze rational functions using an intuitive approach to limits and synthesize functions by considering compositions and inverses. Students expand their work with trigonometric functions and their inverses and complete the study of the conic sections begun in Mathematics 10. They enhance their understanding of probability by considering probability distributions. Previous experiences with series are augmented. High School Math 12 is appropriate for those students that complete Math 11 STEM.

**High School STEM Readiness Mathematics**  
This course is designed for students who have completed the Math 11 (LA) course and subsequently decided they are interested in pursuing a STEM career. It includes standards that would have been covered in Math 11 (STEM) but not in Math 11 (LA) (i.e. standards in the CCSS document that are marked with a “+”), selected topics from the suggested CCSS Math 12 course, and topics drawing from standards covered in Math 9 and Math 10 as needed for coherence.

**Advanced Mathematical Modeling**  
Students continue to build upon their algebra and geometry foundations and expand their understanding through further mathematical experiences. The primary focal points of Advanced Mathematical Modeling include the analysis of information using statistical methods and probability, modeling change and mathematical relationships, mathematical decision making in finance, and spatial and geometric modeling for decision-making. Students learn to become critical consumers of the quantitative data that surround them every day, knowledgeable decision makers who use logical reasoning and mathematical thinkers who can use their quantitative skills to solve problems related to a wide range of situations. As they solve problems in various applied situations, they develop critical skills for success in college and careers, including investigation, research, collaboration and both written and oral communication of their work. As students work with these topics, they continually rely on mathematical processes, including problem-solving techniques, appropriate mathematical language and communication skills, connections within and outside mathematics and reasoning. Students also use multiple representations, technology, applications and modeling and numerical fluency in problem-solving contexts.

**Transition Mathematics for Seniors**  
Transition Math for Seniors prepares students for their entry-level credit-bearing liberal studies mathematics course at the post-secondary level. This course will solidify their quantitative literacy by enhancing numeracy and problem solving skills as they investigate and use the fundamental concepts of algebra, geometry, and introductory trigonometry. *See current policy for placement criteria.

Other Available Courses that have not been described here: AP Calculus, AP Computer Science, AP Statistics, Calculus, Math 9 Lab (Must be taken during 9th grade year), Technical Readiness Mathematics 11 & 12, Other College Level Mathematics courses.
## Transition Plans 2012-2017

### Transitioning to the Next Generation Math Standards

Students will take mathematics annually in grades 9-12. The following chart will assist with scheduling students during transition years (2012-2017) to the Next Generation High School Mathematics Courses:

<table>
<thead>
<tr>
<th>High School Mathematics</th>
<th>Electives Required to Be Offered</th>
<th>Optional Electives</th>
</tr>
</thead>
</table>
| **Transition year 2012-2013** | Algebra I or Math 9  
Algebra II  
Algebra III  
Geometry or Applied Geometry  
Pre-Calculus  
Trigonometry  
Conceptual Mathematics  
Transition Mathematics for Seniors | Calculus  
Probability and Statistics  
Mathematics College Courses  
AP Mathematics Courses  
Algebra Support or Math 9 Lab  
Algebra Support or Math 9 Lab |
| **Transition year 2013-2014** | Algebra I or Math 9  
Algebra II  
Algebra III  
Geometry or Applied Geometry or Math 10  
Pre-Calculus  
Trigonometry  
Conceptual Mathematics  
Transition Mathematics for Seniors | Calculus  
Probability and Statistics  
Mathematics College Courses  
AP Mathematics Courses  
Algebra Support or Math 9 Lab |
| **2014-2015**  
Required new course (Math 9) | Math 9  
Geometry or Math 10  
Algebra II or Math 11 LA, Math 11 STEM, or Math 11 TR  
Algebra III  
Pre-Calculus  
Trigonometry  
Conceptual Mathematics  
Transition Mathematics for Seniors | Math 9 Lab  
Math 12  
AP Calculus  
Advanced Mathematical Modeling  
AP Statistics  
STEM Readiness Mathematics  
AP Computer Science  
Calculus  
Math 12 Technical Readiness  
Other College Level Mathematics Courses |
| **2015-2016**  
Required new course (Math 10) | Math 9  
Math 10  
Algebra II or Math 11 LA, Math 11 STEM, Math 11 TR  
Algebra III  
Pre-Calculus  
Trigonometry  
Transition Mathematics for Seniors | Math 9 Lab  
Math 12  
AP Calculus  
Advanced Mathematical Modeling  
AP Statistics  
STEM Readiness Mathematics  
AP Computer Science  
Calculus  
Math 12 Technical Readiness  
Other College Level Mathematics Courses |
| **2016-2017**  
Required new courses (Math 11 STEM, Math 11 LA, Math 11 TR) | Math 9  
Math 10  
Math 11 LA, Math 11 STEM, Math 11 TR  
Transition Mathematics for Seniors | Math 9 Lab  
Math 12  
AP Calculus  
Advanced Mathematical Modeling  
AP Statistics  
STEM Readiness Mathematics  
AP Computer Science  
Calculus  
Math 12 Technical Readiness  
Other College Level Mathematics Courses |
High School Math 9 Lab Credit
Toward Graduation

Mathematics taught in the ninth grade year is often referred to as “gatekeeper” content to higher level mathematics. Struggling ninth grade students may benefit from a Math 9 Lab experience that is responsive to their individual academic needs through a data driven decision making process. Because some of the highest priority content for college and career readiness comes from Grades 6-8, the Math 9 Lab experiences should address the standards for mathematical practice and connect to the Math 9 content standards while including powerfully useful proficiencies such as applying ratio reasoning in real-world and mathematical problems, computing fluently with positive and negative fractions and decimals, and solving real-world and mathematical problems involving angle measure, area, surface area, and volume. Upon successful completion, students enrolled in a Math 9 Lab course will receive one mathematics credit toward graduation.

The Southern Regional Education Board (SREB) has created high school mathematics online modules that may prove beneficial for use during math lab.

Possible Sequences During Transition

Recommended course sequences listed here are not inclusive. These are not complete lists of every possible pathway to graduation.

<table>
<thead>
<tr>
<th>8th Grade</th>
<th>9th Grade</th>
<th>10th Grade</th>
<th>11th Grade</th>
<th>12th Grade</th>
</tr>
</thead>
<tbody>
<tr>
<td>Algebra I</td>
<td>Math 10 (readiness assessment placement)</td>
<td>Math 11 STEM Or Math 11 LA</td>
<td>AP Calculus Or AP Statistics Or other courses</td>
<td>AP Statistics Or other 4th course options</td>
</tr>
<tr>
<td>(high school credit)</td>
<td>Math 9 (readiness assessment placement)</td>
<td>Math 10</td>
<td>Math 11</td>
<td>Math 12</td>
</tr>
<tr>
<td>8th grade Mathematics</td>
<td>Algebra I (readiness assessment placement)</td>
<td>Math 10</td>
<td>Math 11 STEM</td>
<td>AP Calculus</td>
</tr>
<tr>
<td>8th grade Mathematics</td>
<td>Algebra I (readiness assessment placement)</td>
<td>Math 10</td>
<td>Math 11 STEM/Math 12 (block schedule)</td>
<td>AP Calculus</td>
</tr>
<tr>
<td>8th grade Mathematics</td>
<td>Algebra I (readiness assessment placement)</td>
<td>Math 9</td>
<td>Math 10</td>
<td>Math 11 STEM</td>
</tr>
</tbody>
</table>

Other 4th course options (not limited to):

- AP Computer Science
- Advanced Mathematical Modeling
- Transition Math for Seniors
- STEM Readiness
- Other College Level course
**Possible Course Sequences Upon Complete Transition**

Recommended course sequences listed here are not inclusive. These are not complete lists of every possible pathway to graduation.

<table>
<thead>
<tr>
<th>8th Grade</th>
<th>9th Grade</th>
<th>10th Grade</th>
<th>11th Grade</th>
<th>12th Grade</th>
</tr>
</thead>
<tbody>
<tr>
<td>Math 9 (high school credit)</td>
<td>Math 10</td>
<td>Math 11 STEM or Math 11 LA</td>
<td>AP Calculus or AP Statistics</td>
<td>AP Statistics or other 4th course options</td>
</tr>
<tr>
<td>Math 9 (high school credit)</td>
<td>Math 10</td>
<td>Math 11 STEM</td>
<td>Math 12</td>
<td>AP Calculus</td>
</tr>
<tr>
<td>8th grade Mathematics</td>
<td>Math 9</td>
<td>Math 10</td>
<td>Math 11 STEM</td>
<td>AP Calculus</td>
</tr>
<tr>
<td>8th grade Mathematics</td>
<td>Math 9</td>
<td>Math 10</td>
<td>Math 11 STEM (block schedule)</td>
<td>AP Calculus</td>
</tr>
<tr>
<td>8th grade Mathematics</td>
<td>Math 9</td>
<td>Math 10</td>
<td>Math 11 STEM</td>
<td>4th course option</td>
</tr>
<tr>
<td>8th grade Mathematics</td>
<td>Math 9</td>
<td>Math 10</td>
<td>Math 11 LA</td>
<td>4th course option</td>
</tr>
<tr>
<td>8th grade Mathematics</td>
<td>Math 9</td>
<td>Math 10</td>
<td>Math 11 TR</td>
<td>Math 12 TR</td>
</tr>
<tr>
<td>8th grade Mathematics</td>
<td>Math 9 Lab</td>
<td>Math 10</td>
<td>Math 11 TR</td>
<td>Math 12 TR</td>
</tr>
</tbody>
</table>

Other 4th course options:
- AP Computer Science
- Advanced Mathematical Modeling
- Transition Math for Seniors
- STEM Readiness
- Other College Level courses

Please note that Policies 2510 and 2520.2b are being placed on public comment soon – with State Board approval HS Math course names will change to:

- Math I to Math 9
- Math II to Math 10
- Math III to Math 11
- Math IV to Math 12
Accelerating High School Mathematics Courses

West Virginia’s adoption the Next Generation Mathematics Standards provides a new opportunity to reconsider old practices of accelerating high school mathematics to the middle school. It is strongly recommended that districts systematically consider the full range of issues related to accelerating high school mathematics courses to middle school. Districts should not be rushed or pressured into decisions and should develop a plan along with representative stakeholders, including parents, middle and high school teachers, guidance counselors, and mathematics leaders. Whenever possible, delay course acceleration until the student enters high school.

Here we provide information and resources to ground discussions and decision-making in three inter-related areas of consideration:

• the increased rigor of the grade 8 standards;
• options for high school pathways that accelerate starting in grade 9 to allow students to reach advanced mathematics courses such as Calculus by grade 12 and
• the offering of high school mathematics in middle school to students for which it is appropriate.

Increased Rigor of Grade 8 standards

Success in High School Math 9 is crucial to students’ overall academic success and their continued interest and engagement in mathematics. Based on perceived redundancies in the former standards during the middle grades, districts have increasingly offered the former Algebra I course in Grade 8 to enhance rigor. The new Pre-K-8 standards, however, represent a tight progression of skills and knowledge that is inherently rigorous and designed to provide a strong foundation for success in the new, more advanced, Math 9 course.

The NxGen Grade 8 standards are of significantly higher rigor and more coherent than the former Grade 8 standards. The standards address the foundations of Algebra by including content that was previously part of the Algebra I course standards, such as more in-depth study of linear relationships and equations, a more formal treatment of functions, and the exploration of irrational numbers. The NxGen Grade 8 standards also include geometry standards that relate graphing to algebra in a way that was not explored in the 21st century Grade 8 standards. In addition, the statistics presented in the NxGen Grade 8 standards are more sophisticated than those previously included in middle school and connect linear relations with the representation of bivariate data. The new grade 8 standards address more algebra topics than our previous grade 8 standards.

The High School Math 9 course builds on the Grade 8 standards and is correspondingly more advanced than the old Algebra I course. Because many of the topics previously included in the old Algebra I course are in the new Grade 8 standards, the Next Generation High School Math 9 course starts with more advanced topics and includes more in depth work with linear functions, exponential functions and relationships, and goes beyond the previous high school standards in statistics.

1 The increased placement of eighth graders into Algebra is a national trend, see “The Misplaced Math Student: Lost in Eighth-Grade Algebra” at http://www.brookings.edu/reports/2008/0922_education_loveless.aspx.
The selection and placement of students into accelerated opportunities must be done carefully in order to ensure success. It is recommended that placement decisions be made based upon a set of criteria including a readiness assessment to be reviewed by a team of stakeholders that includes teachers and instructional leadership. It is further recommended, due to the coherence and increased rigor in the middle school mathematics, if possible, to delay course acceleration until the student enters high school. Below are options to consider.

Accelerated High School Pathways

High school mathematics will culminate for many students during 12th grade with courses such as High School Math12 and/or Advanced Mathematical Modeling. Although this would represent a robust and rigorous course of study, some students will seek the opportunity to advance to mathematics courses beyond these courses. The following models are only some of the pathways by which students’ mathematical needs could be met. Districts are encouraged to work with their mathematics leadership, teachers, and curriculum coordinators to design pathways that best meet the abilities and needs of their students.

For students who study the 8th grade standards in grade 8, there are pathways that will lead them to advanced mathematics courses in high school, such as Calculus. In high school, compressed and accelerated pathways may follow these models, among others:

- Students could “double up” by enrolling in the Math 10 course during the same year as Math 11 or “doubling up” Math 11 STEM and Math 12;
- Some very advanced students will be able to move from Math 11 STEM to Calculus. Districts are encouraged to work with their mathematics leadership, teachers, and curriculum coordinators to review the Math 11 STEM course to determine if they desire to add objectives from Math 12 to the Math 11 STEM course to further ensure students success in Calculus.

Note that the accelerated high school pathways delay decisions about which students to accelerate while still allowing access to advanced mathematics in grade 12.

<table>
<thead>
<tr>
<th>9th Grade</th>
<th>10th Grade</th>
<th>11th Grade</th>
<th>12th Grade</th>
</tr>
</thead>
<tbody>
<tr>
<td>Math 9</td>
<td>Math 10</td>
<td>Math 11 STEM</td>
<td>AP Calculus</td>
</tr>
<tr>
<td>Math 9</td>
<td>Math 10</td>
<td>Math 11 STEM/Math 12 (block schedule)</td>
<td>AP Calculus</td>
</tr>
</tbody>
</table>

High School Mathematics in Middle School

Students who have demonstrated the ability to meet the full expectations of the standards quickly should, of course, be encouraged to do so. There are a variety of ways and opportunities for students to advance to mathematics courses. Districts are encouraged to work with their mathematics leadership, teachers, and curriculum coordinators to design an accelerated pathway that best meet the needs of their students. For those students ready to move at a more accelerated pace, one method that has been recommended by the writers of the Common Core State Standards is to compress the standards for any three consecutive grades and/or courses into an accelerated two-year pathway. Students who follow a compacted pathway will be undertaking advanced work at an accelerated pace. This creates a challenge for these students as well as their teachers, who will be teaching within a compressed timeframe 8th Grade standards and Math 9 standards that are significantly more rigorous than in the past.
The Next Generation Mathematics Standards in grades 6-8 are coherent, rigorous, and non-redundant, so the offering of high school coursework in middle school to students for whom it is appropriate requires careful planning to ensure that all content and practice standards are fully addressed (no skipping of critical middle school content). The Common Core State Standards initiative has provided “compacted” pathways in which the standards from Grade 7, Grade 8, and Model Mathematics 9) course could be compressed into an accelerated pathway for students in grades 7 and 8, allowing students to enter the Model Mathematics 10) course in grade 9. The “compacted” pathways can be found in the document Common Core State Standards for Mathematics Appendix A: Designing High School Mathematics Courses Based on the Common Core State Standards, at http://www.corestandards.org/the-standards.

WVDE Math 9 Readiness Assessment

The Office of Instruction and the Office of Assessment and Accountability collaborated to create the WVDE Math 9 Readiness Test, an assessment available on the Acuity platform designed to assist districts in determining the appropriateness of accelerating a student into High School Math 9 during their 8th grade school year.

Data obtainable from this assessment, when used in conjunction with the Math 9 Readiness Indicators Checklist, can be used to inform the proper placement of students desiring to accelerate their math instruction. This assessment does not have a “cut score” and should not be used as the only criterion for acceleration.

The WVDE Math 9 Readiness Test will be shared with Acuity district administrators. Having only been shared with the district administrators, the assessment must either be assigned to the students by the district administrator or shared with the Acuity school administrators to assign.

Questions concerning the use of the data should be directed to Lou Maynus, Office of Instruction; questions concerning the Acuity assessment should be directed to Terri Sappington, Office of Assessment and Accountability.
Office of Instruction  
High School Math 9 Readiness Indicators Checklist

Use the following descriptors to aid in determination of student acceleration to Math 9 in the 8th grade.

<p>| | |</p>
<table>
<thead>
<tr>
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<tbody>
<tr>
<td><strong>Student’s Name</strong></td>
<td></td>
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<tr>
<td><strong>County/School</strong></td>
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</tbody>
</table>

**Current School Year**

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
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<tbody>
<tr>
<td><strong>Mathematics Teacher Recommendation</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Mathematics grades</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Mathematics Summative Test Score (Grade 8 ACT Explore, WESTEST 2, Smarter Balance)</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Reading Summative Test Score</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Parent/Student Recommendations</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Math 9 Readiness Assessment score</strong></td>
<td></td>
</tr>
</tbody>
</table>

**Previous School Year**

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mathematics grades</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Mathematics Summative Test score (Grade 8 ACT Explore, WESTEST 2, Smarter Balance)</strong></td>
<td></td>
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</tbody>
</table>

**Placement Recommendation and Comments:**

<p>| | |</p>
<table>
<thead>
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</thead>
<tbody>
<tr>
<td><strong>Recommendation and Comments</strong></td>
<td></td>
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</tbody>
</table>

Signature/Title  

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Signature/Title  

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Students enrolled in Transition Math for Seniors take the COMPASS Mathematics Test at the end of the course. Upon reviewing last year’s compass scores, we found the adaptive assessment do not permit students with deficits in the Pre-Algebra/Numerical Skills to move on to the higher levels of mathematics in the Algebra portion of the assessment which are taught in the Transition Math for Seniors course. Teachers may need to incorporate the Pre-Algebra/Numerical Skills in their coursework for specific students by revisiting the following skills:

- Pre-Algebra/Numerical Skills: Operations with integers, operations with fractions, operations with decimals, exponents, square roots, scientific notation, ratios, proportions, percentages and averages

Students start with the Pre-Algebra Test. If the student performs well enough, the student will continue automatically to the Algebra Test:

- Algebra: substituting values, setting up equations, basic operations with polynomials, linear equations with one variable, exponents with radicals, rational expressions and linear equations with two variables.

Students may benefit from reviewing the types of questions they will encounter on the Compass test. Sample questions are provided at [http://www.act.org/compass/sample/index.html](http://www.act.org/compass/sample/index.html). Johnson County Community College’s mathematics department has developed an online practice for the COMPASS Mathematics test ([http://www.jccc.edu/testing/math-placement.html](http://www.jccc.edu/testing/math-placement.html)). The online practice provides students with immediate feedback about their responses. If the answer is incorrect, the student is provided a brief explanation, a tutorial video and the message to try again. Other sources include:

- Wright State University Prepare for Mathematics Test [http://www.wright.edu/placement/math.html](http://www.wright.edu/placement/math.html)
- City University of New York (CUNY) Mathematics Practice Questions [http://www.cuny.edu/academics/testing/cuny-assessment-tests/resources/A_Sample_of_The_CUNY_Assessment_Test_in_Mathematics.pdf](http://www.cuny.edu/academics/testing/cuny-assessment-tests/resources/A_Sample_of_The_CUNY_Assessment_Test_in_Mathematics.pdf)
- Pre-Algebra Online Tests [http://www.hostos.cuny.edu/oaa/compass/prealgebra.htm](http://www.hostos.cuny.edu/oaa/compass/prealgebra.htm)
- Algebra Online Tests [http://www.hostos.cuny.edu/oaa/compass/algebra.htm](http://www.hostos.cuny.edu/oaa/compass/algebra.htm)
- Brazosport College
- Pre-Algebra and Algebra learning modules with a video demonstration and practice problems [http://www.brazosport.edu/StudentSuccessCenter/Pages/Algebra-Modules.aspx](http://www.brazosport.edu/StudentSuccessCenter/Pages/Algebra-Modules.aspx)

For questions related to the mathematics content, email me at [lmaynus@access.k12.wv.us](mailto:lmaynus@access.k12.wv.us). Other questions concerning the Compass Mathematics Test administration should be addressed to Dr. Beth Cipoletti at [dcipolet@access.k12.wv.us](mailto:dcipolet@access.k12.wv.us).
High Leverage Mathematics Instruction Practices

These are nine research-affirmed instructional practices that correlate with high levels of student achievement and that should be incorporated into all mathematics instruction at all levels.

<table>
<thead>
<tr>
<th>Practices</th>
<th>Comments/Observations/Reflections</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Effective teachers of mathematics respond to most student answers with “why?”, “how do you know that?”, or “can you explain your thinking?”</td>
<td></td>
</tr>
<tr>
<td>2. Effective teachers of mathematics conduct daily cumulative review of critical and prerequisite skills and concepts at the beginning of every lesson.</td>
<td></td>
</tr>
<tr>
<td>3. Effective teachers of mathematics elicit, value, and celebrate alternative approaches to solving mathematics problems so that students are taught that mathematics is a sense-making process for understanding why and not memorizing the right procedure to get the one right answer.</td>
<td></td>
</tr>
<tr>
<td>4. Effective teachers of mathematics provide multiple representations – for example, models, diagrams, number lines, tables and graphs, as well as symbols – of all mathematical work to support the visualization of skills and concepts.</td>
<td></td>
</tr>
<tr>
<td>5. Effective teachers of mathematics create language-rich classrooms that emphasize terminology, vocabulary, explanations and solutions.</td>
<td></td>
</tr>
<tr>
<td>6. Effective teachers of mathematics take every opportunity to develop number sense by asking for, and justifying, estimates, mental calculations and equivalent forms of numbers.</td>
<td></td>
</tr>
<tr>
<td>7. Effective teachers of mathematics embed the mathematical content they are teaching in contexts to connect the mathematics to the real world.</td>
<td></td>
</tr>
<tr>
<td>8. Effective teachers of mathematics devote the last five minutes of every lesson to some form of formative assessments, for example, an exit slip, to assess the degree to which the lesson’s objective was accomplished.</td>
<td></td>
</tr>
<tr>
<td>9. Effective teachers of mathematics demonstrate through the coherence of their instruction that their lessons – the tasks, the activities, the questions and the assessments – were carefully planned.</td>
<td></td>
</tr>
</tbody>
</table>

These nine research-based instructional practices are high yield practices. This document can be used for self reflection, peer observation, principal and teacher professional development.
http://wvde.state.wv.us/instruction/math.html

Grade Level Descriptive Analysis documents for the Next Generation Mathematics Content Standards and Objectives have been designed to help West Virginia educators provide standards based mathematics instruction. The purpose of the documents are to increase student achievement by ensuring educators understand specifically what the new standards mean a student must know, understand, and are able to do. These documents may be used to facilitate discussions among teachers and curriculum staff and to encourage coherence in the sequence, pacing, and units of study for grade-level curricula. These documents, along with on-going professional development, are resources used to understand and teach the Next Generation Mathematics Content Standards.

See High School Math 10 Example Attached.
Descriptive Analysis of Math 10 Objectives

Grade level specific examples can be found at http://wvde.state.wv.us/instruction/math.html

Descriptive Analysis of the Objective – a narrative of what the child knows, understands and is able to do upon mastery. Teachers should use the information provided in conjunction with the standards and clusters to inform classroom instruction.

### Extending the Number System

**Extend the properties of exponents to rational exponents.**

<table>
<thead>
<tr>
<th>M.2HS.ENS.1</th>
<th>Students will be able to explain orally or in written format a working definition of rational expressions based upon integer exponent laws found in Math 1.</th>
</tr>
</thead>
</table>
|             | Definition of $\frac{1}{n}$: If $n$ is a positive integer greater than 1 and $\sqrt[n]{a}$ is a real number, then $a^{\frac{1}{n}} = \sqrt[n]{a}$.
|             | Teachers should relate rational exponents to integer and whole number exponents. |

<table>
<thead>
<tr>
<th>M.2HS.ENS.2</th>
<th>Convert radical notation to rational exponent notation, and vice-versa.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Rewrite the properties of integer exponents to rational exponents and use them to simplify expressions. This is the application and reinforcement of laws previously learned.</td>
</tr>
<tr>
<td></td>
<td>Examples: $\sqrt[3]{5} = 5^{\frac{1}{3}}$; $5^3 = \sqrt[3]{5^3}$.</td>
</tr>
<tr>
<td></td>
<td>Rewrite using fractional exponents: $\sqrt[10]{5} = 5^{\frac{1}{10}}$.</td>
</tr>
<tr>
<td></td>
<td>Rewrite $\frac{\sqrt{3}}{2}$ in at least three alternate forms. Solution: $\frac{\sqrt{3}}{2} = \frac{1}{\sqrt[3]{3}} = \frac{1}{x^{\frac{3}{2}}}$.</td>
</tr>
<tr>
<td></td>
<td>Rewrite $\frac{\sqrt{3}}{2}$ using only rational exponents.</td>
</tr>
<tr>
<td></td>
<td>Solution: $(2^{-4})^\frac{1}{2} = 2^{-4} = \frac{1}{2}$.</td>
</tr>
<tr>
<td></td>
<td>Rewrite $\sqrt[3]{(x + 1)^3}$ in simplest form.</td>
</tr>
<tr>
<td></td>
<td>Solution: $(x + 1)^{\frac{1}{3}} = x + 1$.</td>
</tr>
<tr>
<td></td>
<td>Be sure to compare contexts where radical form is preferable to rational exponents and vice versa.</td>
</tr>
<tr>
<td>Use properties of rational and irrational numbers.</td>
<td></td>
</tr>
<tr>
<td>-----------------------------------------------------------------------------------------------------------------</td>
<td></td>
</tr>
<tr>
<td><strong>M.2HS.ENS.3</strong> explain why sums and products of rational numbers are rational, that the sum of a rational number and an irrational number is irrational and that the product of a nonzero rational number and an irrational number is irrational. Connect to physical situations, e.g., finding the perimeter of a square of area 2.</td>
<td></td>
</tr>
<tr>
<td>Closure in mathematics occurs when you have a set of things, such as a set of numbers, and when you do an operation on two numbers, the number that results from the operation is from the same set as the original two numbers.</td>
<td></td>
</tr>
<tr>
<td>Introduce this with the concept of why is an odd times an odd number odd. Why is an even number times and odd number, even? Why is an even number times an even number, even?</td>
<td></td>
</tr>
<tr>
<td>Since every difference is a sum and every quotient is a product, this includes differences and quotients as well. Explaining why the four operations on rational numbers produce rational numbers can be a review of students understanding of fractions and negative numbers. Explaining why the sum of a rational and an irrational number is irrational, or why the product is irrational, includes reasoning about the inverse relationship between addition and subtraction (or between multiplication and addition).</td>
<td></td>
</tr>
<tr>
<td>Is a rational times a rational closed under addition? multiplication? - yes</td>
<td></td>
</tr>
<tr>
<td>Is an irrational times an irrational closed under addition? multiplication? - no</td>
<td></td>
</tr>
<tr>
<td>A student could explore sums and products of rational and irrational numbers to discover patterns where the results are either rational or irrational.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Perform arithmetic operations with complex numbers.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>M.2HS.ENS.4</strong> know there is a complex number i such that ( i^2 = -1 ), and every complex number has the form ( a + bi ) with ( a ) and ( b ) real.</td>
</tr>
<tr>
<td>Students need to realize that ( i = \sqrt{-1} ) and understand that the set of complex numbers includes the set of all real numbers and the set of imaginary numbers and has the format of ( a + bi ) where ( a ) and ( b ) are real numbers.</td>
</tr>
<tr>
<td>Examples:</td>
</tr>
<tr>
<td>3 + 4i , 7 = 7 + 0i , 6i = 0 + 6i</td>
</tr>
<tr>
<td>There are in fact two complex square roots of (-1), namely ( i ) and (-i), just as there are two complex square roots of every other real number, except zero, which has one double square root.</td>
</tr>
<tr>
<td>In mathematics, the imaginary unit or unit imaginary number allows the real number system ( R ) to be extended to the complex number system ( C ), which in turn provides at least one root for every polynomial ( P(x) ) (see algebraic closure and fundamental theorem of algebra). The imaginary unit is most commonly denoted by ( i ). The imaginary unit’s core property is that ( i^2 = -1 ). The term “imaginary” is used because there is no real number having a negative square.</td>
</tr>
<tr>
<td>You may want to connect imaginary solutions to the graphs of quadratic functions.</td>
</tr>
<tr>
<td>Examples:</td>
</tr>
<tr>
<td>Within which number system can ( x^2 = -2 ) be solved? Explain how you know.</td>
</tr>
</tbody>
</table>
use the relation \( i^2 = -1 \) and the commutative, associative and distributive properties to add, subtract and multiply complex numbers. Limit to multiplications that involve \( i^2 \) as the highest power of \( i \).

Example:

Simplify the following expression. Justify each step using the commutative, associative and distributive properties.

Solutions may vary; one solution follows:

\[
(3 - 2i)(-7 + 4i)
\]

\[
3(-7 + 4i) - 2i(-7 + 4i) \quad \text{Distributive Property}
\]

\[
-21 + 12i - 8i^2 \quad \text{Distributive Property}
\]

\[
-21 + (12i + 14i) - 8i^2 \quad \text{Associative Property}
\]

\[
-21 + i(12 + 14) - 8i^2 \quad \text{Distributive Property}
\]

\[
-21 + 26i - 8i^2 \quad \text{Computation}
\]

\[
-21 + 26i - 8(-1) \quad i^2=\ -1
\]

\[
-21 + 26i + 8 \quad \text{Computation}
\]

\[
-21 + 8 + 26i \quad \text{Commutative Property}
\]

\[
-13 + 26i \quad \text{Computation}
\]

### Perform arithmetic operations on polynomials.

Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract and multiply polynomials.

Focus on polynomial expressions that simplify to forms that are linear or quadratic in a positive integer power of \( x \).

The addition/subtraction of polynomials as well as the multiplication of polynomials using the distributive property is closed, meaning the result is still a polynomial.

Algebra tiles or other manipulatives for addition, subtraction, and multiplication of polynomials may be used.

Equality: Two complex numbers are equal if and only if their real parts are respectively equal. \( a + bi = c + di \) if \( a = c \) and \( b = d \).

Addition: To add two complex numbers, add the real parts to one another and the imaginary parts to one another. (Subtraction can be defined as addition of the opposite, where the opposite of a complex number is merely the complex number in which the real and the imaginary parts are opposite in sign from their original expression.)

\[
(a + bi) + (c + di) = (a + c) + (b + d)i
\]

Multiplication: Use the distributive property to multiply. Be sure to simplify \( i^2 \).

\[
(a + bi)(c + di) = (ac) + (ad)i + (bc)i + (bd)i^2
\]

Examples:

Try to find two polynomials whose sum/product is not a polynomial.

\[
(3 + 4i) + (5 + 6i) = (3 + 5) + (4 + 6)i = 8 + 10i
\]

\[
(6 + 7i)(5 - 3i) = (6 \cdot 5) + (6 \cdot -3)i + (7 \cdot 5)i + (7 \cdot -3)i^2 = 30 - 18i + 35i + 21 = 51 + 17i
\]
Interpret functions that arise in applications in terms of a context.

**M.2HS.QFM.1**

for a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive or negative; relative maximums and minimums; symmetries; and end behavior. Conversely, the student will use key features of an algebraic function to graph the function.

The student will distinguish linear, quadratic, and exponential relationships based on equations, tables, and verbal descriptions. Given a function in a table or in graphical form, the student will identify key features such as x- and y-intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; and end behavior. Conversely, the student will use key features of an algebraic function to graph the function. The following are examples of appropriate tasks.

**Skill-Based Task:**

Find the maximum height of the path of an arrow modeled by the function \( h(t) = -16t^2 + 96t \). During what interval is the arrow going up? Going down? When does it hit the ground?

**Problem Task:**

Create a situation that could have produced the given data. Use appropriate vocabulary and key features to tell the story.

<table>
<thead>
<tr>
<th>Time</th>
<th>( f(t) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>300</td>
</tr>
<tr>
<td>5</td>
<td>777.5</td>
</tr>
<tr>
<td>10</td>
<td>1010</td>
</tr>
<tr>
<td>15</td>
<td>997.5</td>
</tr>
<tr>
<td>20</td>
<td>740</td>
</tr>
<tr>
<td>25</td>
<td>237.5</td>
</tr>
</tbody>
</table>

Create a situation that could have produced the given data. Use appropriate vocabulary and key features to tell the story.

**M.2HS.QFM.2**

relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function \( h(n) \) gives the number of person-hours it takes to assemble \( n \) engines in a factory, then the positive integers would be an appropriate domain for the function.

The student will identify domains of functions given a graph and identify a domain in a particular context. The following are examples of appropriate tasks.

**Skill-Based Task:**

If a function describes the area of an enclosure made with 100 ft. of fence, what would be an appropriate domain for the function?

**Problem Task:**

Describe a context where the domain of the function would be:
- all real numbers
- whole numbers
- rational numbers
- integers
- even numbers from 2 to 10, inclusive

**M.2HS.QFM.3**

calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. Focus on quadratic functions; compare with linear and exponential functions studied in Mathematics I.

The student will calculate the rate of change in a quadratic function over a given interval from a table or equation. They will compare rates of change in quadratic functions with those in linear or exponential functions. The following are examples of appropriate tasks.

**Skill-Based Task**

Given the function \( f(x) = x^2 - 11x + 24 \), find and interpret the average rate of change over various intervals such as \([0,3]\), \([4,7]\), and \([6,8]\).

**Problem Task:**

A potato is launched into the air. Use rates of change over different intervals to describe the flight of the potato.
### Analyze functions using different representations.

**M.2HS.QFM.4**

Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

- a. Graph linear and quadratic functions and show intercepts, maxima, and minima.
- b. Graph square root, cube root and piecewise-defined functions, including step functions and absolute value functions. Compare and contrast absolute value, step and piecewise defined functions with linear, quadratic, and exponential functions. Highlight issues of domain, range and usefulness when examining piecewise-defined functions.

The student will graph quadratic functions expressed in various forms by hand and using technology to model quadratic functions, when appropriate. The student will graph and find key features of square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. They will find real-world contexts that motivate the use of piecewise-defined functions. The following are examples of appropriate tasks.

**Skill-Based Task:**

Graph the function and identify the key features.

\[ f(x) = x + 2 \] when \( x < 1 \) or \( x = 2 \) and \[ f(x) = x^2 - 3 \] when \( x > 1 \)

**Problem Task:**

Write and graph three different functions whose minimum is \((-1, 5)\).

---

**M.2HS.QFM.5**

Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

- a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values and symmetry of the graph and interpret these in terms of a context.
- b. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as 

\[ y = (1.02)^t, y = (0.97)^t, \]

\[ y = (1.01)^{12}, y = (1.2)^{10}, \]

and classify them as representing exponential growth or decay. M.2HS.QFM.5b extends the work begun in Mathematics I on exponential functions with integer exponents.

The student will factor quadratics and complete the square to find intercepts, extreme values, and symmetry of the graph. They will transition between different forms of quadratic functions and identify the advantages of each. They will extend properties with integer exponents from Math I to properties of exponential functions whose domain includes real numbers. The following are examples of appropriate tasks.

**Skill-Based Task:**

Factor the expression: \( 2x^2 - 9x - 5 \). Transform \( f(x) = x^2 + x - 12 \) into another form to identify the zeros. Transform \( f(x) = x^2 + x - 12 \) into another form to identify the vertex.

**Problem Tasks:**

You are the head of the marketing department at Harmonix, and have just determined your revenue can be modeled by the function \( r(x) = -10x^2 + 100x - 210 \), where \( x \) is the amount spent on advertising in thousands of dollars. Determine an advertising budget.

Your parents offer you two allowance deals

1. $30 plus $2 increase each month, or
2. $20 plus 11% increase each month.

Which deal would you choose and why?
M.2HS.QFM.6
compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum. Focus on expanding the types of functions considered to include, linear, exponential and quadratic. Extend work with quadratics to include the relationship between coefficients and roots and that once roots are known, a quadratic equation can be factored.

The student will compare intercepts, maxima and minima, rates of change, and end behavior of two functions, where one is represented algebraically, graphically, numerically in tables, or by verbal descriptions, and the other is modeled using a different representation. The following are examples of appropriate tasks.

Skill-Based Task:
Which has a greater average rate of change over the interval [5, 10]? 

<table>
<thead>
<tr>
<th>time</th>
<th>ft</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
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<td>740</td>
</tr>
<tr>
<td>25</td>
<td>237.5</td>
</tr>
</tbody>
</table>

Problem Task:
Represent two quadratic functions with a minimum of (0, 2), one expressed in function notation and the other in a table.

Build a function that models a relationship between two quantities.

M.2HS.QFM.7
write a function that describes a relationship between two quantities.

a. determine an explicit expression, a recursive process or steps for calculation from a context.
b. combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model. Focus on situations that exhibit a quadratic or exponential relationship.

Given a linear, exponential, or quadratic context, the student will find an explicit algebraic expression or series of steps to model the context with mathematical representations. The students will combine linear, exponential, quadratic functions using addition subtraction, or multiplication. The following are examples of appropriate tasks.

Skill-Based Task:
The total revenue for a company is found by multiplying the price per unit by the number of units sold minus the production cost. The price per unit is modeled by \( p(n) = -0.5n^2 + 6 \). The number of units sold is \( n \). Production cost is modeled by \( c(n) = 3n + 7 \). Write the revenue function.

Problem Task:
Write a function that models the total number of toothpicks for the \( n \)th term of the pattern. Justify your work.
**Build new functions from existing functions.**

<table>
<thead>
<tr>
<th>M.2HS.QFM.8</th>
<th>M.2HS.QFM.9</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Guidance Document for Secondary Mathematics</strong></td>
<td>The student will determine whether or not a function has an inverse, and find the inverse if it exists. Furthermore, the student will understand that creating an inverse of a quadratic function requires a restricted domain. The following are examples of appropriate tasks.</td>
</tr>
<tr>
<td><strong>Guidance Document for Secondary Mathematics</strong></td>
<td><strong>Skill-Based Task:</strong></td>
</tr>
<tr>
<td><strong>M.2HS.QFM.8</strong></td>
<td><strong>Find inverse functions:</strong></td>
</tr>
<tr>
<td><strong>M.2HS.QFM.8</strong></td>
<td><strong>a. solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. For example, f(x) = 2 x^2 or f(x) = (x + 1)/(x - 1) for x ≠ 1. Focus on linear functions but consider simple situations where the domain of the function must be restricted in order for the inverse to exist, such as f(x) = x^2, x &gt; 0.</strong></td>
</tr>
<tr>
<td><strong>M.2HS.QFM.8</strong></td>
<td><strong>Skill-Based Task:</strong></td>
</tr>
<tr>
<td><strong>M.2HS.QFM.8</strong></td>
<td><strong>Find the inverse of each function, if it exists.</strong></td>
</tr>
<tr>
<td><strong>M.2HS.QFM.8</strong></td>
<td><strong>f(x) = (2x - 3) / 5</strong></td>
</tr>
<tr>
<td><strong>M.2HS.QFM.8</strong></td>
<td><strong>g(x) = 3x^2</strong></td>
</tr>
<tr>
<td><strong>M.2HS.QFM.8</strong></td>
<td><strong>Give an example of a function that does not have an inverse function and explain how you know it does not.</strong></td>
</tr>
<tr>
<td><strong>M.2HS.QFM.8</strong></td>
<td><strong>Problem Task:</strong></td>
</tr>
<tr>
<td><strong>M.2HS.QFM.8</strong></td>
<td><strong>Prove that the inverse of a non-horizontal linear function is also linear and that the slopes are reciprocals.</strong></td>
</tr>
<tr>
<td><strong>M.2HS.QFM.8</strong></td>
<td><strong>Problem Task:</strong></td>
</tr>
<tr>
<td><strong>M.2HS.QFM.8</strong></td>
<td><strong>Graph the following on the same set of axes. Describe the effect of the number 3 in each case.</strong></td>
</tr>
<tr>
<td><strong>M.2HS.QFM.8</strong></td>
<td><strong>f(x) = x^2</strong></td>
</tr>
<tr>
<td><strong>M.2HS.QFM.8</strong></td>
<td><strong>f(x) = -x^2</strong></td>
</tr>
<tr>
<td><strong>M.2HS.QFM.8</strong></td>
<td><strong>f(x) = 3x^2</strong></td>
</tr>
<tr>
<td><strong>M.2HS.QFM.8</strong></td>
<td><strong>f(x) = (x + 3)^2</strong></td>
</tr>
<tr>
<td><strong>M.2HS.QFM.8</strong></td>
<td><strong>f(x) = x^2 - 3</strong></td>
</tr>
<tr>
<td><strong>M.2HS.QFM.8</strong></td>
<td><strong>Problem Task:</strong></td>
</tr>
<tr>
<td><strong>M.2HS.QFM.8</strong></td>
<td><strong>Sort the following functions into the categories even, odd, and neither. Justify your work. For any function in the “neither” category, describe how you could transform it (if possible) into an even or odd function.</strong></td>
</tr>
<tr>
<td><strong>M.2HS.QFM.8</strong></td>
<td><strong>f(x) = x + 3</strong></td>
</tr>
<tr>
<td><strong>M.2HS.QFM.8</strong></td>
<td><strong>i(x) = 5x</strong></td>
</tr>
<tr>
<td><strong>M.2HS.QFM.8</strong></td>
<td><strong>h(x) = (x - 4)^2</strong></td>
</tr>
<tr>
<td><strong>M.2HS.QFM.8</strong></td>
<td>**g(x) = 2</td>
</tr>
<tr>
<td><strong>M.2HS.QFM.8</strong></td>
<td><strong>m(x) = -7x^2</strong></td>
</tr>
<tr>
<td><strong>M.2HS.QFM.8</strong></td>
<td><strong>p(x) = 2x</strong></td>
</tr>
<tr>
<td><strong>M.2HS.QFM.8</strong></td>
<td><strong>Skill-Based Task:</strong></td>
</tr>
<tr>
<td><strong>M.2HS.QFM.8</strong></td>
<td><strong>Graph the following on the same set of axes. Describe the effect of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. Focus on quadratic functions and consider including absolute value functions.</strong></td>
</tr>
<tr>
<td><strong>M.2HS.QFM.8</strong></td>
<td><strong>Problem Task:</strong></td>
</tr>
<tr>
<td><strong>M.2HS.QFM.8</strong></td>
<td><strong>The student will perform transformations on quadratic and absolute value functions with and without technology and describe the effect of each transformation. Given the graph of a function, they will describe all transformations using specific values of k. The student will recognize which transformations take away the even nature of a quadratic or absolute value function. The following are examples of appropriate tasks.</strong></td>
</tr>
</tbody>
</table>
### Construct and compare linear, quadratic, and exponential models and solve problems.

<table>
<thead>
<tr>
<th>M.2HS.QFM.10</th>
<th>The student will use a table to observe that increasing exponential functions eventually grow more quickly than quadratic functions. They will use a graph to observe that exponential functions will grow more quickly than quadratic functions. The following are examples of appropriate tasks.</th>
</tr>
</thead>
<tbody>
<tr>
<td>using graphs and tables, observe that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically; or (more generally) as a polynomial function.</td>
<td>Skill-Based Task: Graph the functions ( y = x^2 ) and ( y = 2^x ) on the same coordinate axes. Compare the values of the functions over various intervals.</td>
</tr>
<tr>
<td>Problem Task: Find a quadratic and exponential function that:</td>
<td>do not intersect</td>
</tr>
<tr>
<td>intersect once</td>
<td>intersect twice</td>
</tr>
<tr>
<td>intersect more than twice</td>
<td></td>
</tr>
</tbody>
</table>

### Applications of Probability

#### Understand independence and conditional probability and use them to interpret data.

<table>
<thead>
<tr>
<th>M.2HS.AOP.1</th>
<th>Define a sample space and events within the sample space. Identify subsets from sample space given defined events, including unions, intersections and complements of events.</th>
</tr>
</thead>
<tbody>
<tr>
<td>describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes or as unions, intersections or complements of other events (“or,” “and,” “not”).</td>
<td>Using the diagram below – as a universal set, students should be able to identify set A and set B as subsets.</td>
</tr>
<tr>
<td>Intersection: The intersection of two sets A and B is the set of elements that are common to both set A and set B. It is denoted by ( A \cap B ) and is read “A intersection B”.</td>
<td>• ( A \cap B ) in the diagram is {1, 5}</td>
</tr>
<tr>
<td>• this means: BOTH/AND</td>
<td></td>
</tr>
<tr>
<td>Union: The union of two sets A and B is the set of elements, which are in A or in B or in both. It is denoted by ( A \cup B ) and is read “A union B”.</td>
<td>• ( A \cup B ) in the diagram is {1, 2, 3, 4, 5, 7}</td>
</tr>
<tr>
<td>• this means: EITHER/OR/ANY</td>
<td></td>
</tr>
<tr>
<td>• could be both</td>
<td></td>
</tr>
<tr>
<td>Complement: The complement of the set ( A \cup B ) is the set of elements that are members of the universal set U but are not in ( A \cup B ). It is denoted by ( (A \cup B)' )</td>
<td>• ( (A \cup B)' ) in the diagram is {8}</td>
</tr>
</tbody>
</table>
### M.2HS.AOP.2
Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities and use this characterization to determine if they are independent.

Identify two events as independent or not. Explain properties of independence.

Compare and contrast the probabilities of two events to determine if the events are independent.

**Multiplication Rule 1:**

When two events, A and B, are independent, the probability of both occurring is: \( P(A \text{ and } B) = P(A) \cdot P(B) \)

Two events, A and B, are independent if the fact that A occurs does not affect the probability of B occurring. Examples are:

- Landing on heads after tossing a coin AND rolling a 5 on a single 6-sided die.
- Choosing a marble from a jar AND landing on heads after tossing a coin.
- Choosing a 3 from a deck of cards, replacing it, AND then choosing an ace as the second card.
- Rolling a 4 on a single 6-sided die, AND then rolling a 1 on a second roll of the die.

Why are drawing a spade and drawing a jack not independent events?

To find the probability of two independent events that occur in sequence, find the probability of each event occurring separately, and then multiply the probabilities. This multiplication rule is defined symbolically below. Note that multiplication is represented by AND.

Students may use spreadsheets, graphing calculators, and simulations to create frequency tables and conduct analyses to determine if events are independent.

### M.2HS.AOP.3
Understand the conditional probability of A given B as \( P(A \text{ and } B)/P(B) \), and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.

Define and calculate conditional probabilities. Know the terminology and symbolism for conditional probabilities.

In general terms, the “conditional probability” \( P(A|B) \) is the probability that A occurs, given that B has occurred and is formally defined as: \( P(A|B) = P(A \text{ and } B) / P(B) \)

Example: \( P(\text{Jack}|\text{Face-card}) \) is the ratio of the chance of both A and B happening to the chance of B happening. In other words, what is the conditional probability of our single card being a Jack, given that we already know that the card is a Face-card?

\[
P(\text{Jack}|\text{Face-card}) = \frac{P(\text{Jack and also a Face-card})}{P(\text{Face-card})}
\]

So we get \( P(\text{Jack}|\text{Face-card}) = \frac{4}{52}/\frac{12}{52} = \frac{1}{3} \)
M.2HS.AOP.4

construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. For example, collect data from a random sample of students in your school on their favorite subject among math, science and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results. Build on work with two-way tables from Mathematics I to develop understanding of conditional probability and independence.

Construct and interpret two-way frequency tables of data for two categorical variables. Calculate probabilities from the table. Use probabilities from the table to evaluate independence of two variables. Use the data to approximate conditional probabilities.

Example:

1. Find the probability that a randomly selected student attends summer school.
2. Find the probability that a student is a boy given that they attend summer school.
3. Find the probability that a randomly selected student is a boy who attends summer school.
4. Are the events “Attending Summer School” and “Boys” independent? Justify your answer.

<table>
<thead>
<tr>
<th>Gender</th>
<th>Summer School</th>
<th>Summer Job</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Girls</td>
<td>25</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>Boys</td>
<td>35</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

M.2HS.AOP.5

recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.

Recognize and explain the concepts of independence and conditional probability in everyday situations.

Example: Practice representing conditional probabilities using tree diagrams. Find the probability that a randomly selected athlete is an honors student.

Example: Create a tree diagram for illustrating the outcomes for a car that has two or four doors and is red, black, or silver. Create questions that can be answered based on the diagram.

Given that you drive a 2-door card, what is the probability that your car is black?
What is the probability that your car is red?
What is the probability that you drive a 2-door vehicle?
Are the number of doors and the color of your car independent events?
**Use the rules of probability to compute probabilities of compound events in a uniform probability model.**

| **M.2HS.AOP 6** | Calculate conditional probabilities using the definition: the conditional probability of A given B as the fraction of B’s outcomes that also belong to A. Interpret the probability in context.  
Find and interpret conditional probabilities using a two-way table, Venn diagram, or tree diagram.  
Find the conditional probability of A given B as the fraction of B’s outcomes that also belong to A and interpret the answer in terms of the model. |
| **M.2HS.AOP 7** | Identify two events as disjoint (mutually exclusive).  
Define the probability of event (A or B) as the probability of their union.  
**Addition Rule:** If A and B are separate sets of events then the probability of either event occurring is  
\[ P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) \]  
Example: There are 125 seniors for this year’s graduating class. The school also has 16 total student council members, 4 from each grade level out of a total student body population of 780. What is the probability that a student selected is a senior or a student council member.  
\[ P(\text{senior or student council}) = \left( P(\text{sr}) + P(\text{sc}) - P(\text{sr and sc}) \right) = \left( \frac{125}{780} + \frac{16}{780} - \frac{4}{780} \right) = \frac{137}{780} \]  
If the events A and B are mutually exclusive, meaning that it is not possible for both sets of events to occur at the same time, then P(A and B)=0 and the addition rule becomes P(A or B) = P(A) + P(B)  
Example: Suppose a high school consists of 25% juniors, 15% seniors, and the remaining 60% are students of other grades. The relative frequency of students who are either juniors or seniors is 40%. We can add the relative frequencies of juniors and seniors because no student can be both a junior and a senior  
\[ P(\text{J or S}) = 0.25 + 0.15 = 0.40 \] |
M.2HS.AOP.8 (+) apply the general Multiplication Rule in a uniform probability model, 
P(A and B) = P(A)P(B | A) = P(B)P(A | B),
and interpret the answer in terms of the model.

In a uniform probability model, calculate probabilities using the General Multiplication Rule. Interpret in context.

The General Multiplication Rule: The probability of two events A and B both happening can be found by

\[ P(A \text{ and } B) = P(A)P(B \mid A). \]

Here \( P(B \mid A) \) is the conditional probability that B occurs, given the information that A has occurred.

From [http://webappa.cdc.gov](http://webappa.cdc.gov)

<table>
<thead>
<tr>
<th>Sex</th>
<th>Number of Deaths</th>
<th>Population</th>
<th>Crude Rate</th>
<th>Age-Adjusted Rate**</th>
</tr>
</thead>
<tbody>
<tr>
<td>Males</td>
<td>2,022</td>
<td>11,303,666</td>
<td>17.89</td>
<td>17.89</td>
</tr>
<tr>
<td>Females</td>
<td>999</td>
<td>10,736,677</td>
<td>9.30</td>
<td>9.30</td>
</tr>
<tr>
<td>Total</td>
<td>3,021</td>
<td>22,040,343</td>
<td>13.71</td>
<td></td>
</tr>
</tbody>
</table>

What is the probability of being a boy in a fatal wreck?

\[ P(\text{boy}) \times P(\text{fatal wreck} \mid \text{boy}) = \]

\[ \frac{11,303,666}{22,040,343} \times \frac{2,022}{11,303,666} = \frac{2,022}{22,040,343} = .0092\% \]

What is the probability of being in a fatal wreck?

\[ \frac{3,021}{22,040,343} = .0137\% \]

Are the events of being a boy and being in a fatal wreck independent events?

The events are not independent because the probability of being a boy in a fatal wreck is not the same as the probability of being in a wreck.
Identify situations as appropriate for use of a permutation or combination to calculate probabilities. Use permutations and combinations in conjunction with probability methods to calculate probabilities of compound events and solve problems.

Fundamental Counting Principle: If one event can occur in \( m \) ways and second event can occur in \( n \) ways, the number of ways the two events can occur in sequence is \( m \times n \). This rule can be extended for any number of events occurring in sequence.

Example: You are purchasing a new vehicle. The possible brands, number of doors, and colors are listed below:

- **Brands**: Chevrolet, Ford, Dodge, Toyota
- **Doors**: 2 door, 4 door
- **Colors**: Black, White, Red, Silver, Blue

How many different ways can you select one brand, number of doors, and one color?

Given the set of ice cream flavors \{chocolate, strawberry, vanilla\}, list all possible two-scoop cones, and find the probability that a randomly selected cone includes chocolate.

**Permutations**: any ordered sequence of \( k \) objects taken from a set of \( n \) distinct objects

**Notation**: \( \binom{n}{r} \)

**Example**: A club of 5 people can elect one member as president and a different member as treasurer. How many different ways can members be elected?

\[
\binom{5}{2} = \frac{5!}{(5-2)!} = \frac{120}{6} = 20
\]

**Combinations**: An unordered subset of \( k \) objects, taken from a set of \( n \) distinct objects is a combination of size \( k \) (“\( n \) choose \( k \)”)  

**Notation**: \( \binom{n}{r} \)

**Example**: If you have a deck of cards without the jokers, how many 5-card hands can you get?

\[
\binom{52}{5} = \frac{52!}{5!(52-5)!} = 2,598,960
\]
### Use probability to evaluate outcomes of decisions.

| M.2HS.AOP.10 (+)                                                                 | Simulate random outcomes using various tools. Analyze the fairness of games by determining the probabilities or the possible outcomes.  
Example: Dice #1 has three 1’s and three 6’s. Dice #2 has two 2’s and four 5’s. When the dice are tossed, the set of dice with the highest number wins. Which set of dice is more likely to win?  
Vicki and Joyce are playing a dice game with two dice. Vicki gets a point if the sum of the numbers on the dice is even, and Joyce gets a point if the sum is odd. Is this game fair? Explain your reasoning.  
They get tired of this game and change the rules. Now Vicki gets a point if the product of the numbers on the dice is even and Joyce gets a point if the product is odd. Is this game fair? Explain your reasoning.  
Resource for random number generator: www.randomnumbergenerator.com |
|---|---|
| M.2HS.AOP.11 (+)                                                                 | Explain in context decisions made based on expected values.  
Judge the reasonableness of mathematical solutions on the basis of the source of the data, the design of the study, the way the data are displayed, and the way the data are analyzed.  
Example: The plot represents the number of points an individual player scores in each of 26 basketball games. At the next game the player scores 10 points. Is this unusual? Use probability to explain your answer. |

This unit sets the stage for work in Mathematics 11, where the ideas of statistical inference are introduced. Evaluating the risks associated with conclusions drawn from sample data (i.e. incomplete information) requires an understanding of probability concepts.
### Similarity, Right Triangle Trigonometry, and Proof

**Understand similarity in terms of similarity transformations**

<table>
<thead>
<tr>
<th>M.2HS.STP.1</th>
<th>Given the following definition:</th>
</tr>
</thead>
<tbody>
<tr>
<td>verify experimentally the properties of dilations given by a center and a scale factor.</td>
<td>A dilation with center ( P ) and scale factor ( r &gt; 0 ) is a mapping ( d ) such that ( d(P) = P ) and for any point ( A ) (different from ( P )), the image ( A' ) lies on line segment ( PA ) and ( PA' = r \cdot PA ).</td>
</tr>
</tbody>
</table>

- a. a dilation takes a line not passing through the center of the dilation to a parallel line and leaves a line passing through the center unchanged.
- b. the dilation of a line segment is longer or shorter in the ratio given by the scale factor.

Students will investigate the dilation of a line from a given point (center of the dilation) to make conjectures about the relationship between the preimage and the image. (In Math 1 students examined rigid transformations taking a preimage to an image.) Students will discover that the image is a line parallel to the preimage.

Note that if the dilation point is not at the origin, a translation can move the dilation to the origin.

The following task from the Illustrative Mathematics Project can be used: Suppose we apply a dilation by a scale factor of 2, centered at the point \( P \), to the figure below.

a. In the picture, locate the images \( A', B', \) and \( C' \) of the points \( A, B, \) and \( C \) under this dilation.

b. Based on your picture in part (a), what do you think happens to the line \( l \) when we perform the dilation?

c. Based on your picture in part (a), what appears to be the relationship between the distance \( A'B' \) and the distance \( AB \)? How about the distances \( B'C' \) and \( BC \)?
M.2HS.STP.2

Given the definition:

Two figures are similar if there exists a sequence of rigid transformations followed by a dilation that maps one figure onto the other.

Students will experimentally apply rigid transformations and dilations to determine if given triangles are similar. They will conjecture by measuring or using dynamic geometry software that there exists a series of rigid transformations and a dilation about the origin that maps one triangle to another if and only if all three pairs of corresponding angles are equal and all three pairs of corresponding sides are in proportion.

The following task from the Illustrative Mathematics Project can be used: In the picture given below, line segments AD and BC intersect at X. Line segments AB and CD are drawn, forming two triangles AXB and CXD.

In each part (a)-(d) below, some additional assumptions about the picture are given. In each problem, determine whether the given assumptions are enough to demonstrate that the two triangles are similar; and if so, what the correct correspondence of vertices is. If the two triangles are similar, verify this result by describing a similarity transformation that maps one triangle onto the other. If not, explain why not.

The lengths AX and XD satisfy the equation 2AX = 3XD.

The lengths AX, BX, CX, and DX satisfy the equation AX/BX = DX/CX.

Lines AB and CD are parallel.

Angle XAB is congruent to angle XCD.
Students will discover shortcuts for determining similarity of triangles. Knowing that the definition of similarity implies that all three pairs of corresponding angles are equal and all three pairs of corresponding sides are in proportion, students will conjecture that not all are necessary to demonstrate similarity.

The following task can be used: Using a variety of tools (patty paper, dynamic geometric software, compass and straightedge), students will create and compare triangles with first one pair of corresponding angles equal, then two pairs. They may also explore what happens when one, two or three pairs of corresponding sides are in proportion. Other interesting combinations might also be considered. They will then make conjectures about which combinations ensure similarity.

Prove geometric theorems.

(Encourage multiple ways of writing proofs, such as in narrative paragraphs, using flow diagrams, in two-column format, and using diagrams without words. Students should be encouraged to focus on the validity of the underlying reasoning while exploring a variety of formats for expressing that reasoning.)

Students will build upon their intuitive understanding of transformations and utilize their visual and spatial thinking to make sense of standard geometric theorems.

The following task can be used: What transformation can be used to demonstrate that vertical angles are congruent? Using a narrative paragraph, a flow diagram, or two–column format, prove that vertical angles are congruent. (From Math 1, students know that angles in a linear pair are supplementary.)

Students will use a transformation to demonstrate that corresponding angles formed by parallel lines are congruent. The formal proof, utilizing a transformational approach, can be easily understood by the student. This result now becomes part of the student’s toolbox.

Proving alternate interior angles congruent follows from this theorem. From Math 1, using congruent triangles, students can prove that points are on a perpendicular bisector of a line segment if and only if they are equidistant from the segment’s endpoints.

From experimentation in M.2HS.STP.1, students conjectured that a dilation takes a line not passing through the center of the dilation to a parallel line and leaves a line passing through the center unchanged. This will now be accepted as a formal postulate.

They will also postulate that “in a dilation of a line segment AB with scale factor r, the length of the image is r AB.”

The following task can be used: What transformation can be used to demonstrate that base angles of an isosceles triangle are congruent? Using a narrative paragraph, a flow diagram, or two–column format, prove that base angles of an isosceles triangle are congruent.
### M.2HS.STP.6

prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other and conversely, rectangles are parallelograms with congruent diagonals. Encourage multiple ways of writing proofs, such as in narrative paragraphs, using flow diagrams, in two-column format and using diagrams without words. Students should be encouraged to focus on the validity of the underlying reasoning while exploring a variety of formats for expressing that reasoning.

Students extend their understanding of parallelograms from Math 1 by developing the properties of this quadrilateral. Students will be taking more responsibility for independently creating their own proofs. These theorems should be conjectured by the student through experimentation with transformations using patty paper, straight edge and compass, or dynamic geometric software before being given as theorems to be proven.

The following task can be used: Construct two segments that bisect each other. Connect their endpoints. What type of quadrilateral is created? Draw a diagram and explain why.

To make sure that a room is rectangular, builders check the two diagonals of the room. Explain what they must check, and why this works.

### Prove theorems involving similarity.

**M.2HS.STP.7**

prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally and conversely; the Pythagorean Theorem proved using triangle similarity.

From experimentation in M.2HS.STP.2, students conjectured that AA and SSS are sufficient to prove triangles similar. The theorem that “The following three statements are equivalent:

1. Two triangles ABC and DEF are similar.
2. Two pairs of corresponding angles are congruent. (AA−)
3. All three pairs of corresponding sides are in proportion. (SSS−)”

can be presented by the teacher.

Using the previously developed tools, students can then prove that a line parallel to one side of a triangle divides the other two proportionally and conversely. Students can prove the Pythagorean Theorem using triangle similarity and also prove its converse.

The following task can be used: A contractor often constructs a triangle with sides 3, 4, and 5 in order to ensure that a right angle is formed. What other sets of three numbers would guarantee that she has a right angle?
Students will apply congruence and similarity in problem solving situations and in extending their ability to develop proofs.

The following task from the Illustrative Mathematics Project can be used: Pablo is practicing bank shots on a standard 4 ft.-by-8 ft. pool table that has a wall on each side, a pocket in each corner, and a pocket at the midpoint of each eight-foot side.

Pablo places the cue ball one foot away from the south wall of the table and one foot away from the west wall, as shown in the diagram below. He wants to bank the cue ball off of the east wall and into the pocket at the midpoint of the north wall.

![Diagram](image)

a. At what point should the cue ball hit the east wall?

b. After Pablo practices banking the cue ball off of the east wall, he tries placing the eight-ball two feet from the east wall, as shown in the diagram below, so that if he shoots the cue ball exactly as he did before, the cue ball will strike the eight-ball directly and sink the eight-ball into the north pocket. How far from the north wall should Pablo place the eight-ball?
### Use coordinates to prove simple geometric theorems algebraically.

<table>
<thead>
<tr>
<th>M.2HS.STP9</th>
<th>Students will compare an analytic proof and a geometric proof for determining the coordinates of a point that partitions a segment in a given ratio.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>The following task can be used: A carpenter has a handrail as shown in the figure. Supports should be placed every 4 inches apart. At what points on the rail should the supports be placed?</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>M.2HS.STP10</th>
<th>Students will develop the definitions of the sine, cosine, and tangent of an acute angle using similar right triangles identifying the sides as opposite, adjacent, or hypotenuse, with respect to the given acute angle.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>The following task can be used: Explain why the tangent of x is the same regardless of which triangle is used in the figure below.</td>
</tr>
</tbody>
</table>

### Define trigonometric ratios and solve problems involving right triangles.

<table>
<thead>
<tr>
<th>M.2HS.STP.12</th>
<th>Students will apply their knowledge of trigonometric ratios and the Pythagorean Theorem to determine distances in realistic situations.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>The following task can be used: Students will determine heights of inaccessible objects using various instruments, such as clinometers, hypsometers, transits, etc.</td>
</tr>
</tbody>
</table>
**Prove and apply trigonometric identities.**

<table>
<thead>
<tr>
<th>M.2HS.STP.13</th>
<th>Students will realize that the distance formula and the Pythagorean identity ( \sin^2(\theta) + \cos^2(\theta) = 1 ) are simply restatements of the Pythagorean Theorem.</th>
</tr>
</thead>
<tbody>
<tr>
<td>prove the Pythagorean identity ( \sin^2(\theta) + \cos^2(\theta) = 1 ) and use it to find ( \sin(\theta) ), ( \cos(\theta) ), or ( \tan(\theta) ), given ( \sin(\theta) ), ( \cos(\theta) ), or ( \tan(\theta) ), and the quadrant of the angle. <strong>In this course, limit ( \theta ) to angles between 0 and 90 degrees. Connect with the Pythagorean theorem and the distance formula. Extension of trigonometric functions to other angles through the unit circle is included in Mathematics 11.</strong></td>
<td></td>
</tr>
</tbody>
</table>

| The following task can be used to apply the Pythagorean identity: Given that \( \sin(\theta) = 4/5 \), find \( \cos(\theta) \). |

---

**Circles With and Without Coordinates**

**Understand and apply theorems about circles.**

<table>
<thead>
<tr>
<th>M.2HS.C.1</th>
<th>The student will prove that all circles are similar.</th>
</tr>
</thead>
<tbody>
<tr>
<td>prove that all circles are similar.</td>
<td>Two geometrical objects are called <strong>similar</strong> if they both have the same shape, or one has the same shape as the mirror image of the other. More precisely, one can be obtained from the other by uniformly scaling (enlarging or shrinking), possibly with additional translation, rotation and reflection. This means that either object can be rescaled, repositioned, and reflected, so as to coincide precisely with the other object. If two objects are similar, each is congruent to the result of a uniform scaling of the other.</td>
</tr>
</tbody>
</table>

**For example,** all circles are similar to each other, all squares are similar to each other, and all equilateral triangles are similar to each other. On the other hand, ellipses are not all similar to each other, nor are hyperbolas all similar to each other.

**Given a circle with radius \( r_1 \) and another circle with radius \( r_2 \) , compare the ratios of the two \( r_1 \), the two diameters, and the two circumferences.**

**Example:** Given a circle of a radius of 3 and another circle with a radius of 5, compare the ratios of the two \( r_1 \), the two diameters, and the two circumferences.

**Prove the two circles are similar:**

- Determine the dilation and transformation.
- Proof options: flowchart, paragraph, and two column
M.2HS.C.2
identify and describe relationships among inscribed angles, rad10 and chords. Include the relationship between central, inscribed and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.
The student will find the measures of central, inscribed, and circumscribed angles of a circle.
The student will show that the measure of the inscribed angle on a diameter is a right angle.
The student will show that the radius of a circle is perpendicular to a tangent line where the radius intersects the circle.
Given the measure of a central angle of a circle is 100 degrees, find the measures of an inscribed angle that intersects the circle at the same points as the central angle.
Why are all inscribed angles that intersect the same points equal regardless of where the vertex is on the circle?

M.2HS.C.3
construct the inscribed and circumscribed circles of a triangle and prove properties of angles for a quadrilateral inscribed in a circle.
The student will inscribe a circle in a triangle.
The student will circumscribe a circle about a triangle.
The student will prove that opposite angles in a quadrilateral inscribed in a circle are supplementary.
Find the unique relationships between the angles of a quadrilateral inscribed within a circle if the quadrilateral is:
- A square.
- A rectangle.
- An isosceles trapezoid.
Find the other two angles.

M.2HS.C.4(+)
construct a tangent line from a point outside a given circle to the circle
Construct a tangent to a circle at a given point on a circle.
Construct a line passing through a point outside the circle that will be tangent to the circle.
### Find arc lengths and areas of sectors of circles.

**M.2HS.C.5**  
derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector. Emphasize the similarity of all circles. Note that by similarity of sectors with the same central angle, arc lengths are proportional to the radius. Use this as a basis for introducing radian as a unit of measure. It is not intended that it be applied to the development of circular trigonometry in this course.

The student will use the concept of similarity to understand that arc length intercepted by a central angle is proportional to the radius.

The student will develop the definition of radians as a unit of measure by relating to arc length.

The student will discover that the measure of the central angle in radians is the constant of proportionality.

The student will derive the formula for the area of a sector.

Complete the table and consider the ratio of arc length to radius for different rad10.

<table>
<thead>
<tr>
<th>Angle</th>
<th>Radius</th>
<th>Arc Length</th>
<th>Arc Lenth/Radius</th>
</tr>
</thead>
<tbody>
<tr>
<td>60 degrees</td>
<td>3 in</td>
<td></td>
<td></td>
</tr>
<tr>
<td>60 degrees</td>
<td>4 in</td>
<td></td>
<td></td>
</tr>
<tr>
<td>60 degrees</td>
<td>5 in</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Construct an arc on a different circle whose length is five times the length of arc AB with the same central angle.

### Translate between the geometric description and the equation for a conic section.

**M.2HS.C.6**  
derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.

The student will use the Pythagorean Theorem to derive the equation of a circle.

The student will find the center of a circle, given its equation.

A circle is tangent to the x-axis and y-axis in the first quadrant. A point of tangency has coordinates (4,0). Find the equation of the circle.

A circle is inscribed in an equilateral triangle.

The equilateral triangle lies in the first quadrant with one vertex at the origin and a second vertex at $(4\sqrt{3},0)$.

Find the equation of the circle.

**M.2HS.C.7**  
derive the equation of a parabola given the focus and directrix.

The student will develop the geometric definition of a parabola.

The student will use the distance formula to derive the equation of a parabola.

Write the equation of a parabola with focus $(3,5)$ and directrix x=-1.

A parabola has focus (-2,1) and directrix y = -3. Determine whether or not the point (2,1) is on the parabola. Justify your response.
### Use coordinates to prove simple geometric theorems algebraically.

<table>
<thead>
<tr>
<th>M.2HS.C.8</th>
<th>The student will use coordinates to prove simple geometric theorems algebraically.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Use coordinates to prove simple geometric theorems algebraically. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, \sqrt{3}) lies on the circle centered at the origin and containing the point ((0, 2)). Include simple proofs involving circles.</td>
<td>Given a circle with center ((x_1, y_1)), determine whether or not the points ((x_1, y_1)) and ((x_2, y_2)) are on the same circle. Justify your response.</td>
</tr>
<tr>
<td>Students explore using these examples: Given a circle with center ((-2,3)), determine whether or not the points ((-4,-1)) and ((3,5)) are on the same circle. Justify your response.</td>
<td>Prove that a triangle with vertices at ((4,3)) ((8,6)) and ((8,3)) is right.</td>
</tr>
</tbody>
</table>

### Explain volume formulas and use them to solve problems.

<table>
<thead>
<tr>
<th>M.2HS.C.9</th>
<th>The student will develop the formulas for the circumference of a circle, area of a circle, and volume of a cylinder, pyramid, and cone using a variety of arguments.</th>
</tr>
</thead>
<tbody>
<tr>
<td>give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri’s principle and informal limit arguments. Informal arguments for area and volume formulas can make use of the way in which area and volume scale under similarity transformations: when one figure in the plane results from another by applying a similarity transformation with scale factor (k), its area is (k^2) times the area of the first.</td>
<td>The student will (understand) use Cavalieri’s Principle to explain/justify solutions.</td>
</tr>
<tr>
<td>The student will use Cavalieri’s Principle to find volumes of solid figures.</td>
<td>Explain why the volume of a cylinder is (V = \pi r^2 h).</td>
</tr>
<tr>
<td>Find the volume of the Great Pyramid of Giza. Use a visual model to represent how to use Cavalieri’s Principle to find the volume of a sphere from the volume of a cone. Give an informal argument referencing Cavalieri’s Principle and relating the volume of a cone to the volume of a sphere. Justify verbally and algebraically.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>M.2HS.C.10</th>
<th>The student will find the volume of cylinders, pyramids, cones, and spheres in contextual problems.</th>
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</thead>
<tbody>
<tr>
<td>use volume formulas for cylinders, pyramids, cones and spheres to solve problems. Volumes of solid figures scale by (k^3) under a similarity transformation with scale factor (k).</td>
<td>Examples:</td>
</tr>
<tr>
<td>Find the volume of a cylindrical oatmeal box.</td>
<td>Given a three-dimensional object, compute the effect on volume of doubling or tripling one or more dimension(s). (For example, how is the volume of a cone affected by doubling the height?)</td>
</tr>
</tbody>
</table>