SUPPORT FOR STANDARDS-BASED INDIVIDUALIZED EDUCATION PROGRAMS: MATHEMATICS K-8, MATH 9

Guidance for West Virginia Schools and Districts

Information in this document was adapted from:

Curriculum Guide To The Alabama Course of Study: Mathematics K-12
Alabama State Department of Education
Special Education Services
P.O. Box 302101
Montgomery, AL 36130
speed@alsde.edu
(334) 242-8114

Grade Level Emphases may be found at http://www.corestandards.org/Math

Notes: This guidance document is available at http://wvde.state.wv.us/osp/

Special thanks to the staff at Hurricane High School in Putnam County for their review of, suggestions for and additions to this document.
# Table of Contents

Acknowledgements ................................................................................................................................. 1

Introduction .................................................................................................................................................. iii

Organization ................................................................................................................................................... iv

Standards for Mathematical Practice .......................................................................................................... vii

Focus Areas K-8 ............................................................................................................................................. x

Math Fluencies K-6 ........................................................................................................................................ x

Mathematics Content Standards, Clusters, Objectives and Instructional Supports Grades K-8, Math 1

- Grade K ....................................................................................................................................................... 1
- Grade 1 ......................................................................................................................................................... 8
- Grade 2 ....................................................................................................................................................... 15
- Grade 3 ....................................................................................................................................................... 23
- Grade 4 ....................................................................................................................................................... 33
- Grade 5 ....................................................................................................................................................... 43
- Grade 6 ....................................................................................................................................................... 54
- Grade 7 ....................................................................................................................................................... 64
- Grade 8 ....................................................................................................................................................... 75
- Math 9 ......................................................................................................................................................... 84

Glossary ......................................................................................................................................................... 95
Introduction


Educators are reminded that content standards indicate minimum content—what all students should know and be able to do by the end of each grade level or course. Local education agencies (LEAs) may have additional instructional or achievement expectations and may provide instructional guidelines that address content sequence, review and remediation.

Support for Standards-Based Individualized Education Programs: Mathematics K-8, Math 9 prepares students for study of the grade-level and content standards through the teaching of prerequisites necessary for learning each content standard. This allows students to work toward grade-level and course content standards while working at individual ability levels. By identifying the prerequisites for each standard, teachers may plan instruction to address the achievement gap experienced by some students with disabilities while still working with all students toward achievement of the same standards.


Educators are encouraged to use the support document to:
- Develop Individualized Education Programs (IEPs).
- Develop lesson plans.
- Plan for Problem-Solving Teams (PSTs) and Support for Personalized Learning (SPL).
- Prepare for Collaborative Teaching Partnerships (CTPs).
- Design tutorials.
- Plan for instructional grouping.
- Plan for parent information and conferences.
- Develop curriculum-based assessments.
- Prepare for state assessments.

DRAFT: Support for Standards-Based Individualized Education Programs: Mathematics K-8, Math 9
Directions for Interpreting the Minimum Required Content

The illustration below are intended to serve as guides for interpreting the Grades K-8, Math 1 minimum required content that all students should learn and be able to do in order to be college- and career-ready. Grades K-8, Math 1 content standards are grouped according to grade-level, standards, clusters and objectives.

**Performance Descriptors** provide five (5) levels of student performance based on mastery of the objectives. In the illustration the performance descriptors are for grade three (3).

**Standards** are large groups of related clusters and objectives. They connect topics and content between and among grade levels. In the illustration the standard is “Number and Operations in Base Ten.”

**Clusters** are groups of related objectives. These identify the primary ideas about what will be learned within the standard. Due to the fact that mathematics is a connected subject, objectives from different clusters may sometimes be closely related. In the example, the cluster is “Use place value understanding and properties of operations to perform multi-digit arithmetic.”

**Objectives** are written beneath each cluster as shown in the following illustration. Objectives define what students should understand (know) and be able to do at the conclusion of a course or grade. Content standards in this document contain minimum required content. The order in which objectives are listed within a course or grade is not intended to convey a sequence for instruction. Each objective completes the phrase “Students will.”

Objectives do not dictate curriculum or teaching methods. For example, one topic may appear before a second in the objectives for a given grade, but this does not necessarily mean that the first must be taught before the second. A teacher might prefer to teach the second topic before the first topic, or might choose to highlight connections by teaching both topics at the same time. In addition, a teacher might prefer to teach a topic of his or her own choosing that leads, as a by-product, to students reaching the objectives for both topics.

**Common Core State Standard Identifiers** are found in the parentheses following each objective. In the illustration for Grade 3, this information identifies the student grade level, the national mathematics Common Core State Standard (CCSS) domain and the CCSS number. For example, the first content standard in the example is followed by content standard identifier (CCSS Math.3.NBT.1) to indicate the student grade level as third (3), the CCSS domain as Number and Operations in Base Ten (NBT), and the CCSS number as one (1).
The system for numbering the Mathematics Next Generation Content Standards and Objective is based on the following:

- **Subject** (Math)
- **Grade Level** (3)
- **Standard** (Number and Operations in Base Ten)
- **Objective** (use place value understanding to round whole numbers to the nearest 10 or 100)

### Grade 3

#### Number & Operations in Base Ten

<table>
<thead>
<tr>
<th>Performance Descriptors</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Distinguished</strong></td>
</tr>
<tr>
<td>Third grade students at the distinguished level in Mathematics: communicate understanding of place value, multiples and properties of operations to justify solutions to real-life problems.</td>
</tr>
<tr>
<td><strong>Above Mastery</strong></td>
</tr>
<tr>
<td>Third grade students at the above mastery level in Mathematics: justify the use of rounding, multiples and the relationship of arithmetic operations when solving real-life problems.</td>
</tr>
<tr>
<td><strong>Mastery</strong></td>
</tr>
<tr>
<td>Third grade students at the mastery level in Mathematics: apply understanding of place value when rounding whole numbers, relate addition and subtraction using properties of operations and multiply one-digit numbers by multiples of ten.</td>
</tr>
<tr>
<td><strong>Partial Mastery</strong></td>
</tr>
<tr>
<td>Third grade students at the partial mastery level in Mathematics: use models to add or subtract.</td>
</tr>
<tr>
<td><strong>Novice</strong></td>
</tr>
<tr>
<td>Third grade students at the novice level in Mathematics: make sense of place value to add, subtract, round or find multiples using tools such as number line or 100 board.</td>
</tr>
</tbody>
</table>

**Cluster: Use place value understanding and properties of operations to perform multi-digit arithmetic.**

- **M.3.NBT.1** use place value understanding to round whole numbers to the nearest 10 or 100. (CCSS Math.3.NBT.1)
- **M.3.NBT.2** fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations and the relationship between addition and subtraction. (CCSS Math.3.NBT.2)
- **M.3.NBT.3** multiply one-digit whole numbers by multiples of 10 in the range 10-90 (e.g., 9 x 80, 5 x 60) using strategies based on place value and properties of operations. (CCSS Math.3.NBT.3)
The organizational components of this guide include objectives, instructional supports, and examples.

**Objectives** are statements that define what all students should know and be able to do at the conclusion of a grade level. Objectives contain minimum required content and complete the phrase “Student will.”

Objectives for a grade level are clearly written, reasonable, measurable, developmentally appropriate and sufficiently rigorous to enable West Virginia students to achieve at levels comparable to other students in the nation and the world. They also provide proportional emphasis to the essential knowledge and processes of a given grade level.

**Instructional Supports** divide the objectives into small instructional units that allow the teacher to break large tasks into smaller tasks to assist the learner gain knowledge within the standard. Instructional supports are useful in lesson planning, classroom instruction and Individualized Education Program (IEP) development. Utilization of instructional supports facilitates having all students working toward grade-level standards while also working at individual ability levels.

**Examples** clarify certain objectives, instructional supports and/or their components. They are illustrative but not exhaustive.
Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices are based on important “processes and proficiencies” that have longstanding importance in mathematics education. The first of these are the National Council of Teachers of Mathematics’ (NCTM) process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council’s report, *Adding It Up: Helping Children Learn Mathematics*. These proficiencies include adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations, and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently, and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own efficacy). The eight Standards for Mathematical Practice are listed below along with a description of behaviors and performances of mathematically proficient students.

Mathematically proficient students:

1. **Make sense of problems and persevere in solving them.** These students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. These students consider analogous problems and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to obtain the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solve complex problems and identify correspondences between different approaches.

2. **Reason abstractly and quantitatively.** Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships. One is the ability to decontextualize, to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents. The second is the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.
3. **Construct viable arguments and critique the reasoning of others.** These students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. These students justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments; distinguish correct logic or reasoning from that which is flawed; and, if there is a flaw in an argument, explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until the middle or upper grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen to or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4. **Model with mathematics.** These students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, students might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, students might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts, and formulas and can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5. **Use appropriate tools strategically.** Mathematically proficient students consider available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a Web site, and use these to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.
6. **Attend to precision.** These students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. Mathematically proficient students are careful about specifying units of measure and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, and express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7. **Look for and make use of structure.** Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see $7 \times 8$ equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as $2 \times 7$ and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. These students also can pause and reflect for an overview and shift perspective. They can observe the complexities of mathematics, such as some algebraic expressions as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers $x$ and $y$.

8. **Look for and express regularity in repeated reasoning.** They notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through $(1, 2)$ with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As students work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details and continually evaluate the reasonableness of their intermediate results.

**Connecting the Standards for Mathematical Practice to the Standards for Mathematical Content**

The eight Standards for Mathematical Practice described on the previous pages indicate ways in which developing student practitioners of the discipline of mathematics increasingly must engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle, and high school years.
The Next Generation Content Standards and Objectives are the Standards for Mathematical Content, are a balanced combination of procedure and understanding. Expectations that begin with the word “understand” are often especially good opportunities to connect mathematical practices to mathematical content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, pause for an overview, or deviate from a known procedure to find a shortcut. Thus, a lack of understanding effectively prevents a student from engaging in the mathematical practices.

In this respect, those content standards which set an expectation of understanding are potential “points of intersection” between the Standards for Mathematical Practice and the Standards for Mathematical Content. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the necessary time, resources, innovative energies, and focus to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.
Priorities in Mathematics in K-8

<table>
<thead>
<tr>
<th>Grade</th>
<th>Focus Areas in Support of Rich Instruction and Expectations of Fluency and Conceptual Understanding</th>
</tr>
</thead>
<tbody>
<tr>
<td>K-2</td>
<td>Addition and subtraction, measurement using whole number quantities</td>
</tr>
<tr>
<td>3-5</td>
<td>Multiplication and division of whole numbers and fractions</td>
</tr>
<tr>
<td>6</td>
<td>Ratios and proportional reasoning; early expressions and equations</td>
</tr>
<tr>
<td>7</td>
<td>Ratios and proportional reasoning; arithmetic of rational numbers</td>
</tr>
<tr>
<td>8</td>
<td>Linear algebra and linear functions</td>
</tr>
</tbody>
</table>

This chart shows the major priority areas in K-8 Math. These are concepts which demand the most time, attention and energy throughout the school year. These are not topics to be checked off a list during an isolated unit of instruction. Rather, these priority areas will be present throughout the school year through rich instructional experiences.

<table>
<thead>
<tr>
<th>Grade</th>
<th>Standard</th>
<th>Required Fluencies</th>
</tr>
</thead>
<tbody>
<tr>
<td>K</td>
<td>K.OA.5</td>
<td>Add/subtract within 5</td>
</tr>
<tr>
<td>1</td>
<td>1.OA.6</td>
<td>Add/subtract within 10</td>
</tr>
<tr>
<td>2</td>
<td>2.OA.2</td>
<td>Add/subtract within 20 (know single-digit sums from memory)</td>
</tr>
<tr>
<td></td>
<td>2.NBT.5</td>
<td>Add/subtract within 100</td>
</tr>
<tr>
<td>3</td>
<td>3.OA.7</td>
<td>Multiply/divide within 100 (know single-digit products from memory)</td>
</tr>
<tr>
<td></td>
<td>3.NBT.2</td>
<td>Add/subtract within 1,000</td>
</tr>
<tr>
<td>4</td>
<td>4.NBT.4</td>
<td>Add/subtract within 1,000,000</td>
</tr>
<tr>
<td>5</td>
<td>5.NBT.5</td>
<td>Multi-digit multiplication</td>
</tr>
<tr>
<td>6</td>
<td>6.NS.2,3</td>
<td>Multi-digit division</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Multi-digit decimal operations</td>
</tr>
</tbody>
</table>

Fluent in the particular objective cited here means “fast and accurate.” It might also help think of fluency in math as similar to fluency in foreign language: when you’re fluent, you flow. Fluent isn’t halting, stumbling, or reversing oneself. The word fluency was used judiciously in the standards to mark the endpoints of progressions of learning that begin with solid underpinnings and then pass upward through stage of growing maturity. Some of these fluency expectations are meant to be mental and others may need pencil and paper. But for each of them, there should be no hesitation in getting the answer with accuracy.
Content Emphases by Cluster — Kindergarten*

Not all of the content in a given grade is emphasized equally in the standards. Some clusters require greater emphasis than the others based on the depth of the ideas, the time that they take to master, and/or their importance to future mathematics or the demands of college and career readiness. In addition, an intense focus on the most critical material at each grade allows depth in learning, which is carried out through the Standards for Mathematical Practice.

To say that some things have greater emphasis is not to say that anything in the standards can safely be neglected in instruction. Neglecting material will leave gaps in student skill and understanding and may leave students unprepared for the challenges of a later grade. The following table identifies the Major Clusters, Additional Clusters, and Supporting Clusters for this grade.

Key: ■ Major Clusters; □ Supporting Clusters; ☐ Additional Clusters

Counting and Cardinality
■ Know number names and the count sequence.
■ Count to tell the number of objects.
■ Compare numbers.

Operations and Algebraic Thinking
■ Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

Number and Operations in Base Ten
■ Work with numbers 11-19 to gain foundations for place value.

Measurement and Data
□ Describe and compare measurable attributes.
■ Classify objects and count the number of objects in categories.

Geometry
□ Identify and describe shapes.
□ Analyze, compare, create, and compose shapes.

*Emphases are given at the cluster level. Refer to the Common Core State Standards for Mathematics for the specific standards that fall within each cluster.
In Kindergarten, instructional time should focus on two critical areas: (1) representing and comparing whole numbers, initially with sets of objects; (2) describing shapes and space. More learning time in Kindergarten should be devoted to number than to other topics.

1. Students use numbers, including written numerals, to represent quantities and to solve quantitative problems, such as counting objects in a set; counting out a given number of objects; comparing sets or numerals; and modeling simple joining and separating situations with sets of objects, or eventually with equations such as $5 + 2 = 7$ and $7 - 2 = 5$. (Kindergarten students should see addition and subtraction equations, and student writing of equations in kindergarten is encouraged, but it is not required.) Students choose, combine and apply effective strategies for answering quantitative questions, including quickly recognizing the cardinalities of small sets of objects, counting and producing sets of given sizes, counting the number of objects in combined sets or counting the number of objects that remain in a set after some are taken away.

2. Students describe their physical world using geometric ideas (e.g., shape, orientation, spatial relations) and vocabulary. They identify, name and describe basic two-dimensional shapes, such as squares, triangles, circles, rectangles and hexagons, presented in a variety of ways (e.g., with different sizes and orientations), as well as three-dimensional shapes such as cubes, cones, cylinders and spheres. They use basic shapes and spatial reasoning to model objects in their environment and to construct more complex shapes.

**Mathematical Practices**

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

<table>
<thead>
<tr>
<th>Grade</th>
<th>Focus Areas in Support of Rich Instruction and Expectations of Fluency and Conceptual Understanding</th>
</tr>
</thead>
<tbody>
<tr>
<td>K-2</td>
<td>Addition and subtraction, measurement using whole number quantities</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Grade</th>
<th>Standard</th>
<th>Required Fluencies</th>
</tr>
</thead>
<tbody>
<tr>
<td>K</td>
<td>K.OA.5</td>
<td>Add/subtract within 5</td>
</tr>
</tbody>
</table>
Counting & Cardinality

<table>
<thead>
<tr>
<th>Performance Descriptors</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Distinguished</strong></td>
</tr>
<tr>
<td>Kindergarten students at</td>
</tr>
<tr>
<td>the distinguished level</td>
</tr>
<tr>
<td>in mathematics:</td>
</tr>
<tr>
<td>write and represent</td>
</tr>
<tr>
<td>numbers beyond 100;</td>
</tr>
<tr>
<td>use multiple strategies</td>
</tr>
<tr>
<td>to count how many</td>
</tr>
<tr>
<td>objects when given</td>
</tr>
<tr>
<td>different arrangements</td>
</tr>
<tr>
<td>and structure and</td>
</tr>
<tr>
<td>justify the strategies</td>
</tr>
<tr>
<td>used;</td>
</tr>
<tr>
<td>compare two two-digit</td>
</tr>
<tr>
<td>numbers using multiple</td>
</tr>
<tr>
<td>strategies and</td>
</tr>
<tr>
<td>justify the use of</td>
</tr>
<tr>
<td>greater than, less than,</td>
</tr>
<tr>
<td>than or equal to</td>
</tr>
<tr>
<td>symbols.</td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>

Know number names and the count sequence.

M.K.CC.1 count to 100 by ones and by tens. (CCSS Math.K.CC.1)
- Count to 50 by ones
- Count to 50 by tens
- Count to 20 by ones
- Count to 10 by ones
- Mimic counting by tens
- Mimic counting by ones

M.K.CC.2 count forward beginning from a given number within the known sequence (instead of having to begin at 1). (CCSS Math.K.CC.2)
- Count forward to 100 from a number between 2 and 50
- Count forward to 100 from a number over 50
- Count forward to 50 from a given number
- Count to 100 by ones
- Mimic counting to 100 by ones

M.K.CC.3 write numbers from 0 to 20. Represent a number of objects with a written numeral 0-20 (with 0 representing a count of no objects). (CCSS Math.K.CC.3)
- Write numbers 0 to 10
- Match numerals to quantity 11 to 20
- Match numerals to quantity 0 to 10
- Recognize written numerals 0 to 20
- Demonstrate one to one correspondence for a group of objects 6 to 20
- Demonstrate one to one correspondence for a group of objects 0 to 5
- Trace numerals 0 to 20
- Make purposeful marks such as lines and circles
Count to tell the number of objects.

M.K.CC.4  understand the relationship between numbers and quantities; connect counting to cardinality
- when counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object,
- understand that the last number name said tells the number of objects counted and the number of objects is the same regardless of their arrangement or the order in which they were counted,
- understand that each successive number name refers to a quantity that is one larger. (CCSS Math.K.CC.4)
  a. When Counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object.
    - Count to 20 by ones
    - Mimic counting objects
  b. Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted
    - Know that the last number tells how many when counting 0 to 5 objects
    - Mimic counting objects up to 20
    - Count to 20 by ones
    - Mimic counting to 20 by ones
  c. Understand that each successive number name refers to a quantity that is one larger
    - Define one larger/one more
    - Count objects in a group and identify total after adding one more
    - Count in sequential order
    - Mimic counting in sequential order

M.K.CC.5  count to answer “how many?” questions about as many as 20 things arranged in a line, a rectangular array, a circle, or as many as 10 things in a scattered configuration; given a number from 1–20, count out that many objects. (CCSS Math.K.CC.5)
- Define how many, all together, and in all
- Demonstrate one to one correspondence
  Example: Point out only one object when counting, and stop counting when all objects have been touched
- Count to 20 by ones

Compare numbers.

M.K.CC.6  identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies. (CCSS Math.K.CC.6)
- Define greater than, less than, and equal to
- Count to 20 by ones
- Count objects up to ten

M.K.CC.7  compare two numbers between 1 and 10 presented as written numerals. (CCSS.Math.K.CC.7)
- Compare numbers 1 to 10 using objects
- Name numerals 1 to 10
- Identify numerals 1 to 10
- Count to 10 by ones

Operations & Algebraic Thinking (OA)

<table>
<thead>
<tr>
<th>Performance Descriptors</th>
<th>Distinguished</th>
<th>Above Mastery</th>
<th>Mastery</th>
<th>Partial Mastery</th>
<th>Novice</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kindergarten students at the distinguished level in mathematics: fluently add and subtract with and without representation and write equations to solve word problems.</td>
<td>Kindergarten students at the above mastery level in mathematics: represent addition and subtraction with and without symbols, create and solve word problems with and without objects or drawings, and write equations.</td>
<td>Kindergarten students at the mastery level in mathematics: represent addition and subtraction within ten (fluently to five), solve word problems, and decompose numbers.</td>
<td>Kindergarten students at the partial mastery level in mathematics: represent addition with objects, fingers, drawings, or role play; represent addition word problems using objects or drawings, and decompose numbers to 5.</td>
<td>Kindergarten students at the novice level in mathematics: represent “how many” with objects, drawings, or role play.</td>
<td></td>
</tr>
</tbody>
</table>
Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

M.K.OA.1 represent addition and subtraction with objects, fingers, mental images, drawings, sounds (e.g., claps), acting out situations, verbal explanations, expressions or equations. (CCSS Math.K.OA.1)
- Define addition as combining groups of objects
- Define subtraction as separating groups of objects
- Represent numbers with objects or drawings
- Separate sets with nine or fewer objects
- Combine objects to form sets up to nine

M.K.OA.2 solve addition and subtraction word problems and add and subtract within 10, e.g., by using objects or drawings to represent the problem. (CCSS Math.K.OA.2)
- Understand key words in addition and subtraction word problems
  - Examples: all together, how many more, how many are left, in all
- Represent numbers with objects or drawings
- Separate sets with nine or fewer objects
- Combine objects to form sets up to nine

M.K.OA.3 decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1). (CCSS Math.K.OA.3)
- Identify plus, minus, and equal signs
- Match numerals to objects or drawings
- Identify numerals 1 to 10
- Count 0 to 10

M.K.OA.4 for any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation. (CCSS Math.K.OA.4)
- Write numerals from 0 to 10
- Represent a given numeral 1 to 10 with objects or drawings
- Count forward from a given number

M.K.OA.5 fluently add and subtract within 5. (CCSS Math.K.OA.5)
- Decompose numbers up to 5 using objects or drawings
- Compose numbers up to 5 using objects or drawings
- Count backward from 5
- Count forward to 5

### Number & Operations in Base Ten

<table>
<thead>
<tr>
<th>Performance Descriptors</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Distinguished</strong></td>
</tr>
<tr>
<td><strong>Above Mastery</strong></td>
</tr>
<tr>
<td><strong>Mastery</strong></td>
</tr>
<tr>
<td><strong>Partial Mastery</strong></td>
</tr>
<tr>
<td><strong>Novice</strong></td>
</tr>
</tbody>
</table>

Work with numbers 11-19 to gain foundations for place value.

M.K.NBT.1 compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation, e.g., 18 = 10 + 8; understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones. (CCSS Math.K.NBT.1)
- Define ones and tens
- Match the number in the ones and tens position to a pictorial representation or manipulative of the value
- Add numbers 1-9 to ten to create teen numbers using manipulative or place value blocks
- Count objects up to 10
Measurement & Data

### Performance Descriptors

<table>
<thead>
<tr>
<th>Distinguished</th>
<th>Above Mastery</th>
<th>Mastery</th>
<th>Partial Mastery</th>
<th>Novice</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kindergarten students at the distinguished level in mathematics: represent and design shapes using measurable attributes; justify the classification of objects and show representation.</td>
<td>Kindergarten students at the above mastery level in mathematics: organize shapes according to measurable attributes; categorize and describe groups of objects.</td>
<td>Kindergarten students at the mastery level in mathematics: describe and compare measurable attributes using vocabulary such as more/less, taller/shorter, etc.; classify, count, and sort objects equal to or less than ten.</td>
<td>Kindergarten students at the partial mastery level in mathematics: recognize some measurable attributes; counts objects by given attributes.</td>
<td>Kindergarten students at the novice level in mathematics: match objects by a given attribute such as big/little, short/tall, etc.; sort objects into groups by a given attribute.</td>
</tr>
</tbody>
</table>

**Describe and compare measurable attributes.**

M.K.MD.1 describe measurable attributes of objects, such as length or weight and describe several measurable attributes of a single object. (CCSS Math.K.MD.1)

- Define length and weight
- Explore objects in relationship to length and weight

M.K.MD.2 directly compare two objects with a measurable attribute in common, to see which object has “more of”/“less of” the attribute, and describe the difference. For example, directly compare the heights of two children and describe one child as taller/shorter. (CCSS Math.K.MD.2)

- Use vocabulary related to length and weight
  Examples: longer, shorter, heavier, lighter
- Identify objects by length and weight
  Examples: shortest pencil, heaviest rock
- Sort objects according to measurable attributes

**Classify objects and count the number of objects in each category.**

M.K.MD.3 classify objects into given categories, count the numbers of objects in each category, and sort the categories by count. Category counts should be limited to less than or equal to 10. (CCSS Math.K.MD.3)

- Identify more and less when given two groups of objects
- Identify object attributes
  Examples: color, shape, size, texture, use
- Count objects up to ten
- Count to 10 by ones

### Geometry

**Performance Descriptors**

<table>
<thead>
<tr>
<th>Distinguished</th>
<th>Above Mastery</th>
<th>Mastery</th>
<th>Partial Mastery</th>
<th>Novice</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kindergarten students at the distinguished level in mathematics: rearrange, draw a representation and describe shapes and their attributes; justify the likeness and differences of two- and three-dimensional shapes.</td>
<td>Kindergarten students at the above mastery level in mathematics: create and design with shapes and describe attributes; relate two- and three-dimensional shapes to shapes within the environment.</td>
<td>Kindergarten students at the mastery level in mathematics: identify, name and describe two- and three-dimensional shapes in the environment, by their orientation, size and relative positions; analyze, compare and describe two- and three-dimensional shapes; model, build and draw shapes, and use simple shapes to compose larger ones.</td>
<td>Kindergarten students at the partial mastery level in mathematics: match, name and describe some two and three dimensional shapes; sort two- and three-dimensional shapes.</td>
<td>Kindergarten students at the novice level in mathematics: match and know the names of some shapes in the environment; match two- and three-dimensional shapes.</td>
</tr>
</tbody>
</table>

**Identify and Describe Shapes** (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders and spheres)

M.K.G.1 describe objects in the environment using names of shapes and describe the relative positions of these objects using terms such as above, below, beside, in front of, behind and next to.(CCSS Math.K.G.1)

- Recognize location and position
  Examples: above, below, beside, in front of, behind, next to
- Identify cubes, cones, cylinders, and spheres
- Imitate actions to place items
- Match shapes
M.K.G.2 correctly name shapes regardless of their orientations or overall size. (CCSS Math.K.G.2)
  - Recognize shapes
  - Sort shapes with different attributes
    Example: sort different size or color squares, circles, triangles, rectangles, or hexagons

M.K.G.3 identify shapes as two-dimensional (lying in a plane, “flat”) or three-dimensional (“solid”). (CCSS Math.K.G.3)
  - Define two-dimensional shapes and three-dimensional shapes
  - Sort flat and solid objects
  - Explore two-dimensional shapes and three-dimensional figures

**Analyze, Compare, Create and Compose Shapes**

M.K.G.4 analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/“corners”) and other attributes (e.g., having sides of equal length). (CCSS Math.K.G.4)
  - Define similar and different
  - Use vocabulary related to two-dimensional shapes and three-dimensional figures
    Examples: vertices (corners), faces (flat surfaces), edges, sides, angles
  - Recognize vocabulary related to two-dimensional shapes and three-dimensional figures
  - Identify two-dimensional shapes and three-dimensional figures
  - Identify shapes

M.K.G.5 model shapes in the world by building shapes from components (e.g., sticks and clay balls) and drawing shapes. (CCSS Math.K.G.5)
  - Recognize attributes of shapes
  - Identify cubes, cones, cylinders, and spheres
  - Identify squares, circles, triangles, rectangles, and hexagons
  - Identify shapes in the environment
  - Trace shapes
  - Make purposeful marks such as lines and circles

M.K.G.6 compose simple shapes to form larger shapes. For example, “Can you join these two triangles with full sides touching to make a rectangle?” (CCSS Math.K.G.6)
  - Combine shapes to fill the area of a given shape
  - Decompose pictures made of simple shapes
  - Recognize shapes
  - Match pieces by color, image, or shape to complete a puzzle
Not all of the content in a given grade is emphasized equally in the standards. Some clusters require greater emphasis than the others based on the depth of the ideas, the time that they take to master, and/or their importance to future mathematics or the demands of college and career readiness. In addition, an intense focus on the most critical material at each grade allows depth in learning, which is carried out through the Standards for Mathematical Practice.

To say that some things have greater emphasis is not to say that anything in the standards can safely be neglected in instruction. Neglecting material will leave gaps in student skill and understanding and may leave students unprepared for the challenges of a later grade. The following table identifies the Major Clusters, Additional Clusters, and Supporting Clusters for this grade.

Key: ■ Major Clusters; □ Supporting Clusters; ☐ Additional Clusters

Operations and Algebraic Thinking
■ Represent and solve problems involving addition and subtraction.
■ Understand and apply properties of operations and the relationship between addition and subtraction.
■ Add and subtract within 20.
■ Work with addition and subtraction equations.

Number and Operations in Base Ten
■ Extending the counting sequence.
■ Understand place value.
■ Use place value understanding and properties of operations to add and subtract.

Measurement and Data
■ Measure lengths indirectly and by iterating length units.
☐ Tell and write time.
■ Represent and interpret data.

Geometry
☐ Reason with shapes and their attributes.

*Emphases are given at the cluster level. Refer to the Common Core State Standards for Mathematics for the specific standards that fall within each cluster.
In Grade 1, instructional time should focus on four critical areas: (1) developing understanding of addition, subtraction and strategies for addition and subtraction within 20; (2) developing understanding of whole number relationships and place value, including grouping in tens and ones; (3) developing understanding of linear measurement and measuring lengths as iterating length units; and (4) reasoning about attributes of and composing and decomposing geometric shapes.

1. Students develop strategies for adding and subtracting whole numbers based on their prior work with small numbers. They use a variety of models, including discrete objects and length-based models (e.g., cubes connected to form lengths), to model add-to, take-from, put-together, take-apart and compare situations to develop meaning for the operations of addition and subtraction and to develop strategies to solve arithmetic problems with these operations. Students understand connections between counting and addition and subtraction (e.g., adding two is the same as counting on two). They use properties of addition to add whole numbers and to create and use increasingly sophisticated strategies based on these properties (e.g., “making tens”) to solve addition and subtraction problems within 20. By comparing a variety of solution strategies, children build their understanding of the relationship between addition and subtraction.

2. Students develop, discuss and use efficient, accurate and generalizable methods to add within 100 and subtract multiples of 10. They compare whole numbers (at least to 100) to develop understanding of and solve problems involving their relative sizes. They think of whole numbers between 10 and 100 in terms of tens and ones (especially recognizing the numbers 11 to 19 as composed of a ten and some ones). Through activities that build number sense, they understand the order of the counting numbers and their relative magnitudes.

3. Students develop an understanding of the meaning and processes of measurement, including underlying concepts such as iterating (the mental activity of building up the length of an object with equal-sized units) and the transitivity principle for indirect measurement.

4. Students compose and decompose plane or solid figures (e.g., put two triangles together to make a quadrilateral) and build understanding of part-whole relationships as well as the properties of the original and composite shapes. As they combine shapes, they recognize them from different perspectives and orientations, describe their geometric attributes and determine how they are alike and different, to develop the background for measurement and for initial understandings of properties such as congruence and symmetry.

Students should apply the principle of transitivity of measurement to make indirect comparisons, but they need not use this technical term.

Mathematical Practices
1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

<table>
<thead>
<tr>
<th>Grade</th>
<th>Focus Areas in Support of Rich Instruction and Expectations of Fluency and Conceptual Understanding</th>
</tr>
</thead>
<tbody>
<tr>
<td>K-2</td>
<td>Addition and subtraction, measurement using whole number quantities</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Grade</th>
<th>Standard</th>
<th>Required Fluencies</th>
</tr>
</thead>
<tbody>
<tr>
<td>K</td>
<td>K.OA.5</td>
<td>Add/subtract within 5</td>
</tr>
<tr>
<td>1</td>
<td>1.OA.6</td>
<td>Add/subtract within 10</td>
</tr>
</tbody>
</table>
Support for Standards-Based Individualized Education Programs: Mathematics K-8, Math 9

Operations & Algebraic Thinking (OA)

<table>
<thead>
<tr>
<th>Performance Descriptors</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Distinguished</strong></td>
</tr>
<tr>
<td>First grade students at the distinguished level in mathematics: represent and justify solutions to addition and subtraction problems; compare and justify the effectiveness of different properties of operations in solving the same equation; explain and justify strategies to calculate fluently to 20; construct and respond to arguments about equivalency in number sentences.</td>
</tr>
</tbody>
</table>

**Represent and solve problems involving addition and subtraction.**

M.1.OA.1 use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart and comparing, with unknowns in all positions, e.g., by using objects, drawings and equations with a symbol for the unknown number to represent the problem. (CCSS Math.1.OA.1)

- Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem
- Understand key words in addition and subtraction word problems
- Examples: sum, difference, all together, how many more, how many are left, in all
- Define subtraction as separating groups of objects, taking from, or taking apart
- Define addition as combining groups of objects, adding to, or putting together
- Represent addition and subtraction with objects, fingers, mental images, drawings, sounds, acting out situations, verbal explanation, expressions, or equations
- Represent numbers with objects or drawings
- Use objects to combine and separate groups

M.1.OA.2 solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings and equations with a symbol for the unknown number to represent the problem. (CCSS Math.1.OA.2)

- Solve addition word problems with sums less than or equal to 10, e.g., by using objects or drawings to represent the problem
- Understand key words in addition word problems
- Define addition as combining groups of objects, adding to, or putting together
- Represent addition with objects, fingers, mental images, drawings, sounds, acting out situations, verbal explanation, expressions, or equations
- Represent numbers with objects or drawings
- Use objects to combine groups

**Understand and apply properties of operations and the relationship between addition and subtraction.**

M.1.OA.3 apply properties of operations as strategies to add and subtract. Examples: If \(8 + 3 = 11\) is known, then \(3 + 8 = 11\) is also known. (Commutative Property of Addition.) To add \(2 + 6 + 4\), the second two numbers can be added to make a ten, so \(2 + 6 + 4 = 2 + 10 = 12\). (Associative Property of Addition.) (Students need not use formal terms for these properties.) (CCSS Math.1.OA.3)

- Define addition and subtraction
- Recognize properties of operations
- Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., \(5 = 2 + 3\) and \(5 = 4 + 1\))
- Apply signs \(+, -, =\) to actions of joining and separating sets
- Identify fact families to ten
- Recognize the value of zero
M.1.OA.4 understand subtraction as an unknown-addend problem. For example, subtract 10 – 8 by finding the number that makes 10 when added to 8. (CCSS Math.1.OA.4)
- Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1)
- Identify fact families to ten
- Recall basic addition facts to ten
- Represent addition and subtraction with objects, fingers, mental images, drawings, sounds, and acting out situation, verbal explanations, expressions, or equations

Add and subtract within 20.
M.1.OA.5 relate counting to addition and subtraction (e.g., by counting on 2 to add 2). (CCSS Math.1.OA.5)
- Represent addition and subtraction with objects, fingers, mental images, drawings, sounds, acting out situations, verbal explanations, expressions, or equations
- Count forward and backward from a given number
- Count to 20 by ones

M.1.OA.6 add and subtract within 20, demonstrating fluency for addition and subtraction within 10 and use strategies such as counting on; making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decomposing a number leading to a ten (e.g., 13 – 4 = 13 – 3 – 1 = 10 – 1 = 9); using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 – 8 = 4); and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13). (CCSS Math.1.OA.6)
- Decompose numbers less than or equal to 10
- Add and subtract within 5
- Count forward and backward from a given number
- Represent addition and subtraction with objects, fingers, mental images, drawings, sounds, acting out situations, verbal explanations, expressions, or equations

Work with addition and subtraction equations.
M.1.OA.7 understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. For example, which of the following equations are true and which are false? 6 = 6, 7 = 8 – 1, 5 + 2 = 2 + 5, 4 + 6 = 5 + 2. (CCSS Math.1.OA.7)
- Define true, false, and equal
- Demonstrate equal using manipulative or object drawing
- Recall basic addition facts to ten
- Recognize equation symbols in vertical and horizontal addition and subtraction problems
- Represent addition and subtraction with objects, fingers, mental images, drawings, sounds, acting out situations, verbal explanations, expressions, or equations

M.1.OA.8 determine the unknown whole number in an addition or subtraction equation relating three whole numbers. For example, determine the unknown number that makes the equation true in each of the equations 8 + ? = 11, 5 = ? – 3, 6 + 6 = ?. (CCSS Math.1.OA.8)
- Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1)
- Identify fact families as a relationship between addition and subtraction
- Recall basic addition and subtraction facts to ten
- Identify plus, minus, and equal signs
- Represent addition and subtraction with objects, fingers, mental images, drawings, sounds, acting out situations, verbal explanations, expressions, or equations
Number & Operations in Base Ten

<table>
<thead>
<tr>
<th>Performance Descriptors</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Distinguished</strong></td>
</tr>
<tr>
<td>First grade students at</td>
</tr>
<tr>
<td>the distinguished level in</td>
</tr>
<tr>
<td>mathematics: represent and explain various counting patterns beyond 120; analyze and justify the reasoning about the value of each digit; create models to illustrate strategies.</td>
</tr>
</tbody>
</table>

Extend the counting sequence.

M.1.NBT.1 count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects with a written numeral.

(CCSS Math.1.NBT.1)
- Write numerals from 0 to 20
- Recognize numerals to 100
- Match the numeral to the number objects or picture of objects
- Count to 100 by ones
- Count to 20 by ones
- Identify and name numerals 1-9
- Trace numerals 1-9

Understand place value.

M.1.NBT.2 understand the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases:

a. 10 can be thought of as a bundle of ten ones — called a “ten.”

b. The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight or nine ones.

c. The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight or nine tens (and 0 ones).

(CCSS Math.1.NBT.2)
- Match the number in the ones and tens position to a pictorial representation or manipulative of the value
- Represent numbers with multiple models
  - Examples: models = base ten blocks, number lines, linking cubes, straw bundles
- Count to 100 by tens
- Count 10 objects
- Count to 10 by ones
- Name numerals 0 to 19

M.1.NBT.3 compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, =, and <.

(CCSS Math.1.NBT.3)
- Define greater than, less than, and equal to
- Demonstrate greater than, less than, and equal to using manipulatives, object drawings, or numbers 0 to 10
- Use comparison vocabulary
  - Examples: greater than, less than, and equal to
- Recognize symbols for greater than, less than, and equal to
- Determine the value of the digits in the ones and tens place
- Identify sets with more, less, or equal objects
- Imitate creating sets of a given size
Use place value understanding and properties of operations to add and subtract.

M.1.NBT.4 add within 100, including adding a two-digit number and a one-digit number and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used and understand that in adding two-digit numbers, one adds tens and tens, ones and ones and sometimes it is necessary to compose a ten. (CCSS Math.1.NBT.4)
- Demonstrate regrouping, total sum, and solve using drawings and concrete models
- Model written method for recording horizontal addition problems
- Determine the value of the number in the ones and tens place
- Match the number in the ones, tens, and hundreds position to a pictorial representation or manipulative of the value
- Represent numbers with multiple models
  Examples: Models – base ten blocks, number lines, linking cubes, straw bundles
- Recall single-digit addition facts

M.1.NBT.5 given a two-digit number, mentally find 10 more or 10 less than the number, without having to count and explain the reasoning used. (CCSS Math.1.NBT.5)
- Define more and less
- Demonstrate conceptual understanding of adding and subtracting 10 using concrete models
- Count backward from 100 by tens
- Count forward to 100 by tens
- Count to 100 by ones

M.1.NBT.6 subtract multiples of 10 in the range 10-90 from multiples of 10 in the range 10-90 (positive or zero differences) using concrete models or drawings and strategies based on place value, properties of operations and/or the relationship between addition and subtraction and relate the strategy to a written method and explain the reasoning used. (CCSS Math.1.NBT.6)
- Demonstrate conceptual understanding of subtracting using concrete models
- Model written method for recording problems involving subtraction of 10 from multiples of 10
- Count backward from 100 by tens
- Count forward to 100 by tens
- Mimic counting to 100 by tens

Measurement & Data

<table>
<thead>
<tr>
<th>Performance Descriptors</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Distinguished</strong></td>
</tr>
<tr>
<td>First grade students at the distinguished level in mathematics: demonstrate and justify the need for a standard unit of measure and make sound decisions about which unit of measure is appropriate; flexibly use time in everyday life; determine multiple ways to represent and interpret the same data.</td>
</tr>
</tbody>
</table>

Measure lengths indirectly and by iterating length units.

M.1.MD.1 order three objects by length and compare the lengths of two objects indirectly by using a third object. (CCSS Math.1.MD.1)
- Define length
- Use vocabulary related to length
  Examples: longer, shorter, longest, shortest, taller
- Identify objects by length
  Examples: shortest pencil, tallest boy
- Sort objects according to length
  Example: sort short pencils from long pencils
- Explore objects in relationship to length

M.1.MD.2 express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end and understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps. (CCSS Math.1.MD.2)
- Describe gap and overlap
- Describe what it means to measure using non-standard units
- Model measuring non-standard units
Tell and write time.

M.1.MD.3 tell and write time in hours and half-hours using analog and digital clocks. (CCSS Math.1.MD.3)
- Describe the short hand as the hour hand and the long hand as the minute hand on an analog clock
- Describe the first number as the hour, and the numbers after the colon as the minutes on a digital clock
- Count to 30 by ones
- Recognize numbers 1 to 12, and 30
- Trace numerals 1 to 12, and 30
- Associate digital and analog clocks with the measurement of time

Represent and interpret data.

M.1.MD.4 organize, represent, interpret data with up to three categories, ask and answer questions about the total number of data points, how many in each category and how many more or less are in one category than in another. (CCSS Math.1.MD.4)
- Define more and less
- Describe methods for representing data
  - Examples: pictographs, tally charts, bar graphs, and Venn diagrams
- Locate information on data displays
- Classify objects into given categories; count the number of objects in each category, and sort the categories by count
- Recognize different types of data displays

Geometry

<table>
<thead>
<tr>
<th>Performance Descriptors</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Distinguished</strong></td>
</tr>
<tr>
<td>First grade students at the distinguished level in mathematics: use nets (paper templates) to construct three-dimensional shapes and partition two-dimensional shapes into sixths and eighths.</td>
</tr>
</tbody>
</table>

Reason with Shapes and Their Attributes

M.1.G.1 distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g., color, orientation, overall size), build and draw shapes to possess defining attributes. (CCSS Math.1.G.1)
- Define side, angle, closed, and open
- Describe attributes of shapes
  - Examples: number of sides, number of angles
- Identify two-dimensional shapes
- Sort two-dimensional shapes
- Identify basic attributes
  - Examples: color, shape, size

M.1.G.2 compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones and right circular cylinders) to create a composite shape and compose new shapes from the composite shape. (Students do not need to learn formal names such as "right rectangular prism.") (CCSS Math.1.G.2)
- Combine shapes to fill in the area of a given shape
- Replicate composite shapes
- Decompose pictures made of simples shapes
- Name shapes
  - Examples: square, circle, triangle, rectangle, and hexagon
- Recognize shapes

M.1.G.3 partition circles and rectangles into two and four equal shares, describe the shares using the words halves, fourths and quarters and use the phrases half of, fourth of and quarter of, describe the whole as two of, or four of the shares and understand for these examples that decomposing into more equal shares creates smaller shares. (CCSS Math.1.G.3)
- Define halves, quarters, whole, parts (shares), and equal
- Demonstrate sharing situations to show equal smaller shares
- Distinguish between equal and non-equal parts
- Decompose pictures made of simple shapes
- Identify squares, circles, triangles, and rectangles
Content Emphases by Cluster — Grade 2*

Not all of the content in a given grade is emphasized equally in the standards. Some clusters require greater emphasis than the others based on the depth of the ideas, the time that they take to master, and/or their importance to future mathematics or the demands of college and career readiness. In addition, an intense focus on the most critical material at each grade allows depth in learning, which is carried out through the Standards for Mathematical Practice.

To say that some things have greater emphasis is not to say that anything in the standards can safely be neglected in instruction. Neglecting material will leave gaps in student skill and understanding and may leave students unprepared for the challenges of a later grade. The following table identifies the Major Clusters, Additional Clusters, and Supporting Clusters for this grade.

Key: ■ Major Clusters; □ Supporting Clusters; □ Additional Clusters

Operations and Algebraic Thinking
■ Represent and solve problems involving addition and subtraction.
■ Add and subtract within 20.
■ Work with equal groups of objects to gain foundations for multiplication.

Number and Operations in Base Ten
■ Understand place value.
■ Use place value understanding and properties of operations to add and subtract.

Measurement and Data
■ Measure and estimate lengths in standard units.
■ Relate addition and subtraction to length.
■ Work with time and money.
■ Represent and interpret data.

Geometry
□ Reason with shapes and their attributes.

*Emphases are given at the cluster level. Refer to the Common Core State Standards for Mathematics for the specific standards that fall within each cluster.
In Grade 2, instructional time should focus on four critical areas: (1) extending understanding of base-ten notation; (2) building fluency with addition and subtraction; (3) using standard units of measure; and (4) describing and analyzing shapes.

1. Students extend their understanding of the base-ten system. This includes ideas of counting in fives, tens and multiples of hundreds, tens and ones, as well as number relationships involving these units, including comparing. Students understand multi-digit numbers (up to 1000) written in base-ten notation, recognizing that the digits in each place represent amounts of thousands, hundreds, tens, or ones (e.g., 853 is 8 hundreds + 5 tens + 3 ones).

2. Students use their understanding of addition to develop fluency with addition and subtraction within 100. They solve problems within 1000 by applying their understanding of models for addition and subtraction, and they develop, discuss, and use efficient, accurate, and generalizable methods to compute sums and differences of whole numbers in base-ten notation, using their understanding of place value and the properties of operations. They select and accurately apply methods that are appropriate for the context and the numbers involved to mentally calculate sums and differences for numbers with only tens or only hundreds.

3. Students recognize the need for standard units of measure (centimeter and inch) and they use rulers and other measurement tools with the understanding that linear measure involves an iteration of units. They recognize that the smaller the unit, the more iterations they need to cover a given length.

4. Students describe and analyze shapes by examining their sides and angles. Students investigate, describe, and reason about decomposing and combining shapes to make other shapes. Through building, drawing, and analyzing two- and three-dimensional shapes, students develop a foundation for understanding area, volume, congruence, similarity, and symmetry in later grades.

**Mathematical Practices**
1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

<table>
<thead>
<tr>
<th>Grade</th>
<th>Focus Areas in Support of Rich Instruction and Expectations of Fluency and Conceptual Understanding</th>
</tr>
</thead>
<tbody>
<tr>
<td>K-2</td>
<td>Addition and subtraction, measurement using whole number quantities</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Grade</th>
<th>Standard</th>
<th>Required Fluencies</th>
</tr>
</thead>
<tbody>
<tr>
<td>K</td>
<td>K.OA.5</td>
<td>Add/subtract within 5</td>
</tr>
<tr>
<td>1</td>
<td>1.OA.6</td>
<td>Add/subtract within 10</td>
</tr>
<tr>
<td>2</td>
<td>2.OA.2</td>
<td>Add/subtract within 20 (know single-digit sums from memory)</td>
</tr>
<tr>
<td></td>
<td>2.NBT.5</td>
<td>Add/subtract within 100</td>
</tr>
</tbody>
</table>
## Operations & Algebraic Thinking (OA)

<table>
<thead>
<tr>
<th>Performance Descriptors</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Distinguished</strong></td>
</tr>
<tr>
<td>Second grade students at the distinguished level in mathematics:</td>
</tr>
<tr>
<td>analyze givens, constraints, relationships, and goals to plan a solution pathway and justify answers;</td>
</tr>
<tr>
<td>communicate carefully formulated explanations of mental math strategies;</td>
</tr>
<tr>
<td>recognize sums could be expressed as n groups of x objects.</td>
</tr>
</tbody>
</table>

| **Above Mastery**       |
| Second grade students at the above mastery level in mathematics: |
| check answers to problems using a different method and assurance answers are reasonable within the context of the problem; |
| flexibly use mental math strategies; |
| flexibly work with groups of objects representing the sum as repeated addition of equal addends in multiple ways. |

| **Mastery**             |
| Second grade students at the mastery level in mathematics: |
| make sense of quantities, relationships in problem situations, and represent symbolically to solve problem; |
| use various mental strategies and make use of patterns and structures to fluently compute sums and differences; |
| represent the sum of a group of objects as the repeated addition of equal addends. |

| **Partial Mastery**      |
| Second grade students at the partial mastery level in mathematics: |
| solve problems with unknowns in various positions; |
| use mental strategies to add and subtract within 20; |
| write an addition equation to represent the number of objects in a group. |

| **Novice**              |
| Second grade students at the novice level in mathematics: |
| solve one-step word problems by using drawing and concrete materials; |
| add and subtract within 20 using models; |
| determine the total number of objects in a group and whether that number is odd or even. |

### Represent and solve problems involving addition and subtraction.

M.2.OA.1 use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart and comparing, with unknowns in all positions, e.g. by using drawings and equations with a symbol for the unknown number to represent the problem. (CCSS Math.2.OA.1)

- Solve one-step addition and subtraction word problems with an unknown by using drawings and equations with a symbol for the unknown number to represent the problem
- Understand key words in addition and subtraction word problems
  - Examples: adding to, taking from, putting together, taking apart, sum, difference, all together, how many more, how many are left, in all
- Locate the unknown regardless of position
  - Examples: start unknown, change unknown, and result unknown
- Apply signs +, -, = to actions of joining and separating sets
- Solve addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknown in all positions, e.g., by using objects or drawings to represent the problem
- Represent addition and subtraction with objects, fingers, mental images, drawings, sounds, acting out situations, verbal explanations, expressions, or equations
- Represent numbers with objects or drawings

### Add and subtract within 20.

M.2.OA.2 fluently add and subtract within 20 using mental strategies and by end of Grade 2, know from memory all sums of two one-digit numbers. (CCSS Math.2.OA.2)

- Recall single-digit subtraction facts with minuends of 10 or less
- Recall single-digit addition facts with sums up to 10
- Apply addition and subtraction strategies
  - Examples: doubles, doubles plus one, doubles minus one
- Represent addition and subtraction with objects, fingers, mental images, drawings, sounds, acting out situations, verbal explanations, expressions, or equations
- Represent numbers with objects or drawings

### Work with Equal Groups of Objects to Gain Foundations for Multiplication

M.2.OA.3 determine whether a group of objects (up to 20) has an odd or even number of members, e.g. by pairing objects or counting them by 2s and write an equation to express an even number as a sum of two equal addends. (CCSS Math.2.OA.3)

- Define pair, odd, and even
- Recall doubles addition facts with sums up to 20
- Apply signs + and = to actions of joining sets
- Model written method for composing equations
- Skip count by 2s

M.2.OA.4 use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns and write an equation to express the total as a sum of equal addends. (CCSS Math.2.OA.4)

- Distinguish between rows and columns
- Use repeated addition to solve problems with multiple addends
- Count forward in multiples from a given number
  - Examples: 3, 6, 9, 12; 4, 8, 12, 16
- Recall doubles addition facts
- Model written method for composing equations
Number & Operations in Base Ten

<table>
<thead>
<tr>
<th>Performance Descriptors</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Distinguished</strong></td>
</tr>
<tr>
<td>Second grade students at the distinguished level in mathematics:</td>
</tr>
<tr>
<td>communicate precisely and justify to others the meaning of the symbols used in number comparisons with 1,000; calculate fluently and give carefully formulated explanations to justify answers.</td>
</tr>
<tr>
<td><strong>Above Mastery</strong></td>
</tr>
<tr>
<td>Second grade students at the above mastery level in mathematics:</td>
</tr>
<tr>
<td>make sense of quantities within 1,000 and explain how the relationships of the comparisons are reasonable; flexibly use properties of operations when adding and subtracting numbers within 1,000 and use different methods to check answers.</td>
</tr>
<tr>
<td><strong>Mastery</strong></td>
</tr>
<tr>
<td>Second grade students at the mastery level in mathematics:</td>
</tr>
<tr>
<td>make sense of quantities within 1,000 using place value to make comparisons and represent those relationships symbolically; use strategies based on place value, properties of operations and number relationships to add and subtract numbers within 1,000.</td>
</tr>
<tr>
<td><strong>Partial Mastery</strong></td>
</tr>
<tr>
<td>Second grade students at the partial mastery level in mathematics:</td>
</tr>
<tr>
<td>model numbers within 1,000 using base-ten blocks or drawings; or manipulatives; add and subtract within 1,000 using concrete models or drawings.</td>
</tr>
<tr>
<td><strong>Novice</strong></td>
</tr>
<tr>
<td>Second grade students at the novice level in mathematics:</td>
</tr>
<tr>
<td>model numbers within 1,000 using base-ten blocks or drawings; or manipulatives; add and subtract within 1,000 using concrete models or drawings.</td>
</tr>
</tbody>
</table>

Understand place value.

M.2.NBT.1 understand that the three digits of a three-digit number represent amounts of hundreds, tens and ones; e.g. 706 equals 7 hundreds, 0 tens and 6 ones and understand the following as special cases:

a. 100 can be thought of as a bundle of ten tens – called a “hundred.”
b. numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight or nine hundreds (and 0 tens and 0 ones).

(CCSS Math.2.NBT.1)
- Match the number in the ones, tens, and hundreds position to a pictorial representation or manipulative of the value
- Represent numbers with multiple concrete models
  - Examples: concrete models – base ten blocks, number lines, linking cubes, straw bundles
- Count to 1000 by hundreds
- Count to 100 by tens
- Create groups of 10
- Match the numeral in the ones and tens position to a pictorial representation or manipulative of the value
- Match the numeral to the number of objects or picture of objects

M.2.NBT.2 count within 1000 and skip-count by 5s, 10s and 100s. (CCSS Math.2.NBT.2)
- Create a number pattern
- Count backwards from 100 by fives and tens
- Count forward to 100 by fives and tens
- Count to 100 by ones

M.2.NBT.3 read and write numbers to 1000 using base-ten numerals, number names and expanded form. (CCSS Math.2.NBT.3)
- Identify zero as a place holder in two-digit and three-digit numbers
- Match the number in the ones, tens, and hundreds position to a pictorial representation or manipulative of the value
- Identify the value of number in the ones, tens, and hundreds place
- Identify place value for ones, tens, and hundreds
- Read number names one through one hundred
- Write numerals 1 to 100
- Recognize number names one through twenty
- Trace numerals 0 to 100

M.2.NBT.4 compare two three-digit numbers based on meanings of the hundreds, tens and ones digits, using >, = and < symbols to record the results of comparisons. (CCSS Math.2.NBT.4)
- Define greater than, less than, and equal to
- Compare two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, =, and <
- Arrange two-digit numbers in order from greatest to least or least to greatest
- Identify zero as a place holder in two-digit and three-digit numbers
- Model using >, =, and < symbols to record results of comparisons of two-digit numbers
- Select numbers on a number line that are more than, less than, or equal to a specified number
- Match the words greater than, equal to, and less than to the symbols >, =, and <
- Determine the value of the digits in the ones and tens place
- Identify sets with more, less, or equal objects
Use place value understanding and properties of operations to add and subtract.

M.2.NBT.5 fluently add and subtract within 100 using strategies based on place value, properties of operations and/or the relationship between addition and subtraction. (CCSS Math.2.NBT.5)

- Define regrouping, total, sum, difference, and solve
- Represent numbers with multiple models
  - Examples: models – base ten blocks, number lines, linking cubes, straw bundles
- Recall single-digit addition and subtraction facts
- Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten; decomposing a number leading to a ten; using the relationship between addition and subtraction; and creating equivalent but easier or known terms
- Add and subtract within 5
- Match the number in the ones, tens, and hundreds position to a pictorial representation or manipulative of the value

M.2.NBT.6 add up to four two-digit numbers using strategies based on place value and properties of operations. (CCSS Math.2.NBT.6)

- Add within 100, including adding a two-digit number and a one-digit number and adding two-digit numbers, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method, and explain the reasoning used
- Add within 20, demonstrating fluency for addition within 10. Use strategies such as counting on; making ten; decomposing a number leading to a ten; and creating equivalent but easier or known sums
- Determine the value of the number in the ones, tens, and hundreds place
- Model written method for recording horizontal and vertical addition problems
- Understand that the two digits of a two-digit number represent amounts of tens and ones
- Match the number in the ones and tens position to a pictorial representation or manipulative of the value

M.2.NBT.7 add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations and/or the relationship between addition and subtraction, relate the strategy to a written method and understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones and sometimes it is necessary to compose or decompose tens or hundreds. (CCSS Math.2.NBT.7)

- Define regrouping, total, sum, difference, and solve
- Add and subtract two two-digit numbers with and without regrouping
- Determine the value of the number in the ones, tens, and hundreds, and thousands place using concrete models or drawings and strategies based on place value
- Match the number in the ones, tens, hundreds, and thousands position to a pictorial representation or manipulative of the value
- Model written method for recording horizontal and vertical addition and subtraction problems
- Represent two- and three-digit numbers with multiple models
  - Examples: models – base ten blocks, number lines, linking cubes, straw bundles
- Add and subtract within 20, e.g., by using objects or drawings to represent the problem

M.2.NBT.8 mentally add 10 or 100 to a given number 100-900 and mentally subtract 10 or 100 from a given number 100-900. (CCSS Math.2.NBT.8)

- Demonstrate conceptual understanding of adding and subtracting 10 using concrete models
- Recognize the place value of ones, tens, and hundreds
- Count forward and backward by 100
- Count forward and backward by 10
- Recall single-digit subtraction facts
- Recall single-digit addition facts

M.2.NBT.9 explain why addition and subtraction strategies work, using place value and the properties of operations. (Explanations may be supported by drawing or objects.) (CCSS Math.2.NBT.9)

- Explain addition and subtraction problems using concrete objects, pictures
- Use multiple strategies to add and subtract including counting on, counting back, and using doubles
- Recall single-digit subtraction facts
- Recall single-digit addition facts
- Represent addition and subtraction with objects, fingers, mental images, drawings, sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations
### Measurement & Data

<table>
<thead>
<tr>
<th>Performance Descriptors</th>
<th>Distinguished</th>
<th>Above Mastery</th>
<th>Mastery</th>
<th>Partial Mastery</th>
<th>Novice</th>
</tr>
</thead>
<tbody>
<tr>
<td>Second grade students at the distinguished level in mathematics:</td>
<td>Second grade students at the above mastery level in mathematics:</td>
<td>Second grade students at the mastery level in mathematics:</td>
<td>Second grade students at the partial mastery level in mathematics:</td>
<td>Second grade students at the novice level in mathematics:</td>
<td></td>
</tr>
<tr>
<td>provide carefully formulated explanations to justify measurements;</td>
<td>communicate proper use of measurement tools;</td>
<td>estimate length of objects and select the correct tools to accurately measure and compare length;</td>
<td>select appropriate tools to measure and compare lengths of objects;</td>
<td>given the appropriate tool measure the length of objects;</td>
<td></td>
</tr>
<tr>
<td>analyze and justify representations and solutions;</td>
<td>communicate representations and solution;</td>
<td>solve addition and subtraction word problems within 100 involving length and represent quantities of length using number lines and drawings;</td>
<td>add and subtract to solve word problems involving the length of objects;</td>
<td>use concrete models to solve addition word problems involving length of objects;</td>
<td></td>
</tr>
<tr>
<td>tell time to the nearest minute, apply monetary skills to real-world situations and evaluate the reasonableness of their conclusions;</td>
<td>communicate precisely how to accurately tell and write time and create word problems involving money;</td>
<td>accurately tell and write time to the nearest five minutes on analog and digital clocks and solve word problems involving money;</td>
<td>read and write time to the nearest five minutes on digital clocks and recognize the value of coins;</td>
<td>read and write time to the nearest five minutes on digital clocks and recognize the value of coins;</td>
<td></td>
</tr>
<tr>
<td>reason deductively about data showing regularity or trends.</td>
<td>represent and interpret data from various graphs, solve word problems, and communicate important features of data.</td>
<td>collect and represent measurement data up to four categories and solve simple problems through the interpretation of the data presented.</td>
<td>collect measurement data and create multiple representations using at least three categories.</td>
<td>given measurement data, create a simple representation.</td>
<td></td>
</tr>
</tbody>
</table>

#### Measure and estimate lengths in standard units.

**M.2.MD.1** Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks and measuring tapes. (CCSS Math.2.MD.1)
- Identify units of measurement for length
  - Examples: inches, feet, yard; centimeter, meters
- Demonstrate how to use measurement tools
  - Example: avoiding gaps and overlaps
- Identify measurement tools
- Model measuring using non-standard units
- Order three objects by length
- Compare the lengths of two objects indirectly by using a third object
- Describe measurable attributes of objects such as length or weight

**M.2.MD.2** Measure the length of an object twice, using length units of different lengths for the two measurements, describe how the two measurements relate to the size of the unit chosen and compare and contrast plane and solid geometric shapes. (CCSS Math.2.MD.2)
- Identify units of measurement for length
  - Examples: inches, feet, yard; centimeter, meters
- Demonstrate how to use measurement tools
  - Example: avoiding gaps and overlaps
- Identify units of measure on measurement tools
- Use vocabulary related to comparison of length
  - Examples: longer, shorter, longest, shortest, taller
- Identify numbers one to 50

**M.2.MD.3** Estimate lengths using units of inches, feet, centimeters and meters. (CCSS Math.2.MD.3)
- Define estimate
- Measure objects using standard and non-standard units
- Identify units of measure on measurement tools
- Model measuring using non-standard units
- Use vocabulary related to comparison of length
  - Examples: longer, shorter, longest, shortest, taller
- Identify numbers one to 50

**M.2.MD.4** Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit. (CCSS Math.2.MD.4)
- Measure objects using standard units
- Record lengths with appropriate units
- Use subtraction within 20 to solve problems
- Compare length using non-standard units to determine which is longer
- Use vocabulary related to comparison of length
  - Examples: longer, shorter, longest, shortest, and taller
Relate addition and subtraction to length.

M.2.MD.5 use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units, e.g., by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem. (CCSS Math.2.MD.5)

- Use addition and subtraction within 20 to solve one-step addition and subtraction word problems with an unknown number
- Understand key words in addition and subtraction word problems involving length
  Examples: adding to, taking from, putting together, taking apart, sum, difference, all together, how many more, how many are left, in all, inches, feet, yards, longer, shorter, farther, closer
- Locate the unknown number regardless of position
- Add and subtract within 50, e.g., by using objects or drawings to represent the problem
- Model writing equations from word problems
- Identify units of measurement for length
  Examples: inches, feet, yard; centimeter, meter
- Apply signs +, -, = to actions of joining and separating

M.2.MD.6 represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0,1, 2, ... and represent whole-number sums and differences within 100 on a number line diagram. (CCSS Math.2.MD.6)

- Recognize that each successive number name refers to a quantity that is one larger; and each previous number name refers to a quantity that is one less
- Use a number line to add and subtract within 10
- Write numeral 0 to 100
- Trace numerals 0 to 100

Work with time and money.

M.2.MD.7 tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m. (CCSS Math.2.MD.7)

- Tell and write time in hours and half-hours using analog and digital clocks
- Recognize vocabulary terms related to time measurements
  Examples: minute, hour, half hour, o’clock, morning, evening, a.m., p.m.
- Illustrate time to hour and half hour
  Example: given the time 3:00, illustrate long hand and short hand positions on a clock
- Identify the short hand as the hour hand, and the long hand as the minute hand on an analog clock
- Identify the first number as the hour, and the numbers after the colon as the minutes on a digital clock
- Write numerals 0 to 59
- Recognize numerals 0 to 59
- Count to 60 by fives
- Distinguish between analog and digital clocks

M.2.MD.8 solve word problems involving dollar bills, quarters, dimes, nickels and pennies, using $ and ¢ symbols appropriately. Example: If you have 2 dimes and 3 pennies, how many cents do you have? (CCSS Math.2.MD.8)

- Determine the monetary value of a set of like and unlike bills
- Determine the monetary value of a set of like and unlike coins
- Apply addition and subtraction strategies
- Understand key words in addition and subtraction word problems involving money
  Examples: adding to, taking from, putting together, taking apart, sum, difference, all together, how much more, how much left, in all, cents, dollar, change, paid, total
- Count forward from a given number by ones, fives, tens, and twenty-fives
- Identify coins and bills and their value
- Identify symbols for dollar ($), cent (¢)
- Identify coins by name including penny, nickel, dime, and quarter
- Sort pennies, nickels, dimes, and quarters
- Count 10 objects
  Examples: pennies and dollar bills

Represent and interpret data.

M.2.MD.9 generate measurement data by measuring lengths of several objects to the nearest whole unit or by making repeated measurements of the same object and show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units. (CCSS Math.2.MD.9)

- Define length and line plot
- Use vocabulary related to comparison of length
  Examples: longer, shorter, longest, shortest, taller
- Demonstrate rounding up to the nearest whole unit on measurement tools
- Demonstrate measuring length using standard units
- Describe a line plot
- Model measuring length using standard units
- Identify objects by length
- Sort objects according to length
- Explore objects in relationship to length
M.2.MD.10 draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories and solve simple put-together, take-apart and compare problems using information presented in a bar graph. (CCSS Math.2.MD.10)

- Use addition and subtraction within 20 to solve addition and subtraction word problems with an unknown number
- Describe picture graph and bar graph
- Demonstrate conceptual understanding of adding or subtracting using a variety of materials
- Use vocabulary related to comparing data
  Examples: more than, less than, most, least, equal
- Recognize attributes of data displays
- Locate information on data displays
- Classify objects into given categories
- Sort categories by count
- Recognize different types of data displays
- Count objects up to 50

**Geometry**

<table>
<thead>
<tr>
<th>Performance Descriptors</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distinguished</td>
</tr>
<tr>
<td>Second grade students at the distinguished level in mathematics: justify the relationship among shapes and that equal shares of identical wholes do not need to have the same shape.</td>
</tr>
</tbody>
</table>

**Reason with Shapes and Their Attributes**

M.2.G.1 recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces (sizes are compared directly or visually, not compared by measuring) and identify triangles, quadrilaterals, pentagons, hexagons and cubes. (CCSS Math.2.G.1)

- Define side, angle, face, closed, and open
- Use vocabulary related to shape attributes
  Examples: sides, angles, face, closed, open
- Trace shapes
- Sort triangles, quadrilaterals, pentagons, hexagons, and cubes
- Explore triangles, quadrilaterals, pentagons, hexagons, and cubes

M.2.G.2 partition a rectangle into rows and columns of same-size squares and count to find the total number of them. (CCSS Math.2.G.2)

- Define rows, columns, and total
- Identify rectangle
- Count to 20 by ones
- Trace partition in a rectangle

M.2.G.3 partition circles and rectangles into two, three or four equal shares, describe the shares using the words halves, thirds, half of, a third of, etc., describe the whole as two halves, three thirds, four fourths and recognize that equal shares of identical wholes need not have the same shape. (CCSS Math.2.G.3)

- Define halves, thirds, fourths, quarters, whole, parts (shares), and equal
- Distinguish between equal and non-equal parts
- Model partitioning circles and rectangles
- Decompose pictures made of simple shapes
- Identify squares, circles, triangles, and rectangles
- Explore shapes or figures that can be decomposed into smaller equal parts
Mathematics (M) Grade 3

Content Emphases by Cluster — Grade 3*

Not all of the content in a given grade is emphasized equally in the standards. Some clusters require greater emphasis than the others based on the depth of the ideas, the time that they take to master, and/or their importance to future mathematics or the demands of college and career readiness. In addition, an intense focus on the most critical material at each grade allows depth in learning, which is carried out through the Standards for Mathematical Practice.

To say that some things have greater emphasis is not to say that anything in the standards can safely be neglected in instruction. Neglecting material will leave gaps in student skill and understanding and may leave students unprepared for the challenges of a later grade. The following table identifies the Major Clusters, Additional Clusters, and Supporting Clusters for this grade.

Key: ■ Major Clusters; ■ Supporting Clusters; □ Additional Clusters

Operations and Algebraic Thinking
■ Represent and solve problems involving multiplication and division.
■ Understand properties of multiplication and the relationship between multiplication and division.
■ Multiply and divide within 100.
■ Solve problems involving the four operations, and identify and explain patterns in arithmetic.

Number and Operations in Base Ten
□ Use place value understanding and properties of operations to perform multi-digit arithmetic.

Number and Operations—Fractions
■ Develop understanding of fractions as numbers.

Measurement and Data
■ Solve problems involving measurement and estimation of intervals of time, liquid volumes and masses of objects.
■ Represent and interpret data.
■ Geometric measurement: understand concepts of area and relate area to multiplication and to addition.
□ Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.

Geometry
■ Reason with shapes and their attributes.

*Emphases are given at the cluster level. Refer to the Common Core State Standards for Mathematics for the specific standards that fall within each cluster.
Grade Focus Areas in Support of Rich Instruction and Expectations of Fluency and Conceptual Understanding

<table>
<thead>
<tr>
<th>Grade</th>
<th>Focus Areas in Support of Rich Instruction and Expectations of Fluency and Conceptual Understanding</th>
</tr>
</thead>
<tbody>
<tr>
<td>K-2</td>
<td>Addition and subtraction, measurement using whole number quantities</td>
</tr>
<tr>
<td>3-5</td>
<td>Multiplication and division of whole numbers and fractions</td>
</tr>
</tbody>
</table>

In Grade 3, instructional time should focus on four critical areas: (1) developing understanding of multiplication and division and strategies for multiplication and division within 100; (2) developing understanding of fractions, especially unit fractions (fractions with numerator 1); (3) developing understanding of the structure of rectangular arrays and of area; and (4) describing and analyzing two-dimensional shapes.

1. Students develop an understanding of the meanings of multiplication and division of whole numbers through activities and problems involving equal-sized groups, arrays and area models; multiplication is finding an unknown product, and division is finding an unknown factor in these situations. For equal-sized group situations, division can require finding the unknown number of groups or the unknown group size. Students use properties of operations to calculate products of whole numbers, using increasingly sophisticated strategies based on these properties to solve multiplication and division problems involving single-digit factors. By comparing a variety of solution strategies, students learn the relationship between multiplication and division.

2. Students develop an understanding of fractions, beginning with unit fractions. Students view fractions in general as being built out of unit fractions, and they use fractions along with visual fraction models to represent parts of a whole. Students understand that the size of a fractional part is relative to the size of the whole. For example, 1/2 of the paint in a small bucket could be less paint than 1/3 of the paint in a larger bucket, but 1/3 of a ribbon is longer than 1/5 of the same ribbon because when the ribbon is divided into 3 equal parts, the parts are longer than when the ribbon is divided into 5 equal parts. Students are able to use fractions to represent numbers equal to, less than, and greater than one. They solve problems that involve comparing fractions by using visual fraction models and strategies based on noticing equal numerators or denominators.

3. Students recognize area as an attribute of two-dimensional regions. They measure the area of a shape by finding the total number of same-size units of area required to cover the shape without gaps or overlaps, a square with sides of unit length being the standard unit for measuring area. Students understand that rectangular arrays can be decomposed into identical rows or into identical columns. By decomposing rectangles into rectangular arrays of squares, students connect area to multiplication and justify using multiplication to determine the area of a rectangle.

4. Students describe, analyze, and compare properties of two-dimensional shapes. They compare and classify shapes by their sides and angles, and connect these with definitions of shapes. Students also relate their fraction work to geometry by expressing the area of part of a shape as a unit fraction of the whole.

Mathematical Practices
1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

<table>
<thead>
<tr>
<th>Grade</th>
<th>Standard</th>
<th>Required Fluencies</th>
</tr>
</thead>
<tbody>
<tr>
<td>K</td>
<td>K.OA.5</td>
<td>Add/subtract within 5</td>
</tr>
<tr>
<td>1</td>
<td>1.OA.6</td>
<td>Add/subtract within 10</td>
</tr>
<tr>
<td>2</td>
<td>2.OA.2</td>
<td>Add/subtract within 20 (know single-digit sums from memory)</td>
</tr>
<tr>
<td></td>
<td>2.NBT.5</td>
<td>Add/subtract within 100</td>
</tr>
<tr>
<td>3</td>
<td>3.OA.7</td>
<td>Multiply/divide within 100 (know single-digit products from memory)</td>
</tr>
<tr>
<td></td>
<td>3.NBT.2</td>
<td>Add/subtract within 1,000</td>
</tr>
</tbody>
</table>
Represent and solve problems involving multiplication and division.

M.3.OA.1 interpret products of whole numbers, e.g., interpret $5 \times 7$ as the total number of objects in 5 groups of 7 objects each. For example, describe context in which a total number of objects can be expressed as $5 \times 7$. (CCSS Math.3.OA.1)

- Identify and define the parts of a multiplication problem including factors, multiplier, multiplicand, and product
- Use multiplication to find the total number of objects arranged in rectangular arrays based on columns and rows
- Write an equation to express the product of the multipliers (factors)
- Relate multiplication to repeated addition and skip counting
- Apply concepts of multiplication through the use of manipulatives, number stories, skip-counting arrays, area of a rectangle, or repeated addition

Examples: array – 

\[
\begin{array}{cccccccc}
\& \& \& \\
\& \& \& \\
\& \& \& \\
\& \& \& \\
\end{array}
\]

Repeated addition – $8 + 8 + 8 = 24$

- Apply basic multiplication facts through $9 \times 9$ using manipulatives, solving problems, and writing number stories
- Solve addition problems with multiple addends
- Represent addition using manipulatives

M.3.OA.2 interpret whole-number quotients of whole numbers, e.g., interpret $56 \div 8$ as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. For example, describe a context in which a number of shares or a number of groups can be expressed as $56 \div 8$. (CCSS Math.3.OA.2)

- Identify and define the parts of a division problem including divisor, dividend, and quotient
- Model grouping with basic division facts partitioned equally (e.g., $8 \div 2$)
- Recognize division as either repeated subtraction, parts of a set, parts of a whole, or the inverse of multiplication
- Apply properties of operations as strategies to subtract
- Subtract within 20
- Represent equal groups using manipulatives

M.3.OA.3 use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. (CCSS Math.3.OA.3)

- Demonstrate computational understanding of multiplication and division by solving authentic problems with multiple representations using drawings, words, and/or numbers
- Identify key vocabulary words to solve multiplication and division word problems
- Examples: times, every, at this rate, each, per, equal/ equally, in all, total
- Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20 e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem
- Recall basic multiplication facts
- Add and subtract within 20
- Represent repeated addition, subtraction, and equal groups using manipulatives
M.3.OA.4 determine the unknown whole number in a multiplication or division equation relating three whole numbers. For example, determine the unknown number that makes the equation true in each of the equations $8 \times ? = 48$, $5 = ? \div 3$, $6 \times 6 = ?$. (CCSS Math.3.OA.4)

- Use arrays to show equal groups in multiplication and division
- Recall basic multiplication facts
- Determine the unknown whole number in an addition or subtraction equation relating three whole numbers
- Represent repeated addition, repeated subtraction, and equal groups using manipulatives

**Understand properties of multiplication and the relationship between multiplication and division.**

M.3.OA.5 apply properties of operations as strategies to multiply and divide. (Students need not use formal terms for these properties.) Examples: If $6 \times 4 = 24$ is known, then $4 \times 6 = 24$ is also known. (Commutative property of multiplication.) $3 \times 5 \times 2$ can be found by $3 \times 5 = 15$, then $15 \times 2 = 30$, or by $5 \times 2 = 10$, then $3 \times 10 = 30$. (Associative property of multiplication.) Knowing that $8 \times 5 = 40$ and $8 \times 2 = 16$, one can find $8 \times 7$ as $8 \times (5 + 2) = (8 \times 5) + (8 \times 2) = 40 + 16 = 56$. (Distributive property.) (CCSS Math.3.OA.5)

- Define properties of operations
- Apply basic multiplication facts
- Apply properties of operations as strategies to add and subtract
- Count to answer “how many?” questions about as many as 30 things arranged in a rectangular array

M.3.OA.6 understand division as an unknown-factor problem. For example, find $32 \div 8$ by finding the number that makes 32 when multiplied by 8. (CCSS Math.3.OA.6)

- Apply divisibility rules for 2, 5, and 10
  - Example: Recognizing that 32 is divisible by 2 because the digit in the ones place is even
- Apply basic multiplication facts
- Understand subtraction as an unknown-addend problem
- Recognize division as repeated subtraction, parts of a set, parts of a whole, or the inverse of multiplication

**Multiply and divide within 100.**

M.3.OA.7 fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that $8 \times 5 = 40$, one knows $40 \div 5 = 8$) or properties of operations and by the end of Grade 3, know from memory all products of two one-digit numbers. (CCSS Math.3.OA.7)

- Name the first 10 multiples of each one-digit natural number
  - Example: $7, 14, 21, 28, 35, 42, 49, 56, 63, 70$
- Recognize multiplication as repeated addition, and division as repeated subtraction
- Apply properties of operations as strategies to add and subtract
- Recall basic addition and subtraction facts

**Solve problems involving the four operations, and identify and explain patterns in arithmetic.**

M.3.OA.8 solve two-step word problems using the four operations, represent these problems using equations with a letter standing for the unknown quantity and assess the reasonableness of answers using mental computation and estimation strategies including rounding. (This standard is limited to problems posed with whole numbers and having whole number answers; students should know how to perform operations in the conventional order when there are no parenthesis to specify a particular order (Order of Operations).) (CCSS Math.3.OA.8)

- Define the identity property of addition and multiplication
  - Examples: Addition $7 + 0 = 7$, $0 + 7 = 7$
  - Multiplication $450 \times 1 = 450$, $1 \times 450 = 450$
- Estimating sums and differences using multiple methods, including compatible numbers and rounding, to judge the reasonableness of an answer
  - Examples: Compatible numbers $23 + 38$ is approximately $25 + 40$
  - Rounding $286$ is approximately $300$
- Apply commutative, associative, and identity properties for all operations to solve problems
- Identify a rule when given a pattern
- Solve addition and subtraction problems, including word problems, involving one- and two-digit numbers with and without regrouping, using multiple strategies
  - Example: strategies using concrete objects, mental calculations, paper-and-pencil activities
- Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem
- Represent multiplication and division with manipulatives
- Recall basic addition and subtraction facts
M.3.OA.9 identify arithmetic patterns (including patterns in the addition table or multiplication table) and explain them using properties of operations.

For example, observe that 4 times a number is always even and explain why 4 times a number can be decomposed into two equal addends. (CCSS Math.3.OA.9)

- Define arithmetic patterns: geometric or numeric
- Explain arithmetic patterns using properties of operations
  - Example: Observe that 4 times a number is always even, and explain why 4 times a number can be decomposed (separated into parts) into two equal addends
- Recognize arithmetic patterns (including geometric patterns or patterns in the addition table or multiplication table)
- Construct repeating and growing patterns with a variety of representations
- Demonstrate computational fluency, including quick recall, of addition and multiplication facts
- Duplicate an existing pattern
  - Example: Duplicate a numerical or geometric pattern
- Skip count
- Represent addition and multiplication with manipulatives

Number & Operations in Base Ten

<table>
<thead>
<tr>
<th>Performance Descriptors</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Distinguished</strong></td>
</tr>
<tr>
<td>Third grade students at the distinguished level in Mathematics: communicate understanding of place value, multiples and properties of operations to justify solutions to real-life problems.</td>
</tr>
<tr>
<td><strong>Above Mastery</strong></td>
</tr>
<tr>
<td>Third grade students at the above mastery level in Mathematics: justify the use of rounding, multiples and the relationship of arithmetic operations when solving real-life problems.</td>
</tr>
<tr>
<td><strong>Mastery</strong></td>
</tr>
<tr>
<td>Third grade students at the mastery level in Mathematics: apply understanding of place value when rounding whole numbers, relate addition and subtraction using properties of operations and multiply one-digit numbers by multiples of ten.</td>
</tr>
<tr>
<td><strong>Partial Mastery</strong></td>
</tr>
<tr>
<td>Third grade students at the partial mastery level in Mathematics: make sense of place value to add, subtract, round or find multiples using tools such as number line or 100 board.</td>
</tr>
<tr>
<td><strong>Novice</strong></td>
</tr>
<tr>
<td>Third grade students at the novice level in Mathematics: use models to add or subtract.</td>
</tr>
</tbody>
</table>

Use place value understanding and properties of operations to perform multi-digit arithmetic.

M.3.NBT.1 use place value understanding to round whole numbers to the nearest 10 or 100. (CCSS Math.3.NBT.1)

Define rounding

- Round whole numbers from 100 to 999 using whole numbers from 10 to 99
- Model rounding whole numbers to the nearest 100
- Round whole numbers from 10 to 99 using whole numbers from 1 to 9
- Model rounding whole numbers to the nearest 10
- Identify the steps in rounding two- and three-digit numbers
- Example: identify the digit that may change and the number to the right
- Round whole numbers from 1 to 9 and model to show proficiency
- Understand that the two digits of a two-digit number represent amounts of tens and ones
- Match the number in the ones, tens, and hundreds position to a pictorial representation or manipulative of the value

M.3.NBT.2 fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations and/or the relationship between addition and subtraction. (CCSS Math.3.NBT.2)

- Define the commutative and associative properties of addition and subtraction.
- Subtract within 100 using strategies and algorithms based on the relationship between addition and subtraction.
- Subtract within 100 using strategies and algorithms based on properties of operations.
- Add within 100 using strategies and algorithms based on the relationship between addition and subtraction.
- Add within 100 using strategies and algorithms based on properties of operations.
- Add within 100 using strategies and algorithms based on place value.
- Recall basic addition and subtraction facts.

M.3.NBT.3 multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., 9 × 80, 5 × 60) using strategies based on place value and properties of operations. (CCSS Math.3.NBT.3)

- Model place value by multiplying vertically.
- Model properties of operations by multiplying horizontally.
- Recall basic multiplication facts.
- Recall multiplication as repeated addition.
- Apply properties of operations as strategies to add.

Examples: If 8 + 3 = 11 is known, then 3 + 8 = 11 is also known. (Commutative property of addition.) To add 2 + 6 + 4, the second two numbers can be added to make a ten, so 2 + 6 + 4 = 2 + 10 = 12. (Associative property of addition.)
### Number & Operations - Fractions

#### Performance Descriptors

<table>
<thead>
<tr>
<th>Distinguished</th>
<th>Above Mastery</th>
<th>Mastery</th>
<th>Partial Mastery</th>
<th>Novice</th>
</tr>
</thead>
<tbody>
<tr>
<td>Third grade students at the distinguished level in mathematics: justify and communicate the use of fractions to solve real-world problems.</td>
<td>Third grade students at the above mastery level in mathematics: use models or a number line to explain comparisons of fractions and check reasonableness of the comparisons.</td>
<td>Third grade students at the mastery level in mathematics: make sense of fractions as equal parts of a whole or points on a number line, explain equivalent fractions and compare fractions using various criteria and symbols.</td>
<td>Third grade students at the partial mastery level in mathematics: use various models or a number line to compare fractions.</td>
<td>Third grade students at the novice level in mathematics: identify fractions using models.</td>
</tr>
</tbody>
</table>

#### Develop understanding of fractions as numbers.

**M.3.NF.1** understand a fraction 1/b as the quantity formed by 1 part when a whole is partitioned into b equal parts and understand a fraction a/b as the quantity formed by a parts of size 1/b. (CCSS Math.3.NF.1)

- Define fraction, numerator, and denominator.
- Identify the parts of a fraction a/b as the quantity formed by a parts and size 1/b.
- Label numerator, denominator, and fraction bar.
- Identify parts of a whole with two, three, or four equal parts.
- Distinguish between equal and non-equal parts.
- Partition circles and rectangles into two and four equal shares; describe the shares using the words halves, fourths, and quarters; and use the phrases half of, fourth of, and quarter of.

**M.3.NF.2** understand a fraction as a number on the number line and represent fractions on a number line diagram

a. represent a fraction 1/b on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts and recognize that each part has size 1/b and that the endpoint of the part based at 0 locates the number 1/b on the number line.
b. represent a fraction a/b on a number line diagram by marking off a lengths 1/b from 0 and recognize that the resulting interval has size a/b and that its endpoint locates the number a/b on the number line.

(CCSS Math.3.NF.2)

- Recognize fractions as lengths from zero to one.
- Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2…, and represent whole-number sums and differences within 100 on a number diagram.
- Identify a number line.

a. Represent a fraction 1/b on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts.

- Recognize whole numbers as lengths from zero to one.
- Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2…, and represent whole-number sums and differences within 100 on a number diagram.
- Identify a number line.
b. Represent a fraction a/b on a number line diagram by marking off a lengths 1/b from 0. Recognize that the resulting interval has size a/b and that its endpoint locates the number a/b on the number line.

- Label the fractions on a pre-made number line diagram.
- Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2…, and represent whole-number sums and differences within 100 on a number diagram.
- Recognize a number line diagram with equally spaced points.

**M.3.NF.3** explain equivalence of fractions in special cases and compare fractions by reasoning about their size

a. understand two fractions as equivalent (equal) if they are the same size or the same point on a number line,
b. recognize and generate simple equivalent fractions, e.g., 1/2 = 2/4, 4/6 = 2/3 and explain why the fractions are equivalent, e.g., by using a visual fraction model,
c. express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers (Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram.)
d. compare two fractions with the same numerator or the same denominator by reasoning about their size, recognize that comparisons are valid only when the two fractions refer to the same whole, record the results of comparisons with the symbols >, = or < and justify the conclusions, e.g., by using a visual fraction model.
Measurement & Data

Performance Descriptors

<table>
<thead>
<tr>
<th>Distinguished</th>
<th>Above Mastery</th>
<th>Mastery</th>
<th>Partial Mastery</th>
<th>Novice</th>
</tr>
</thead>
<tbody>
<tr>
<td>Third grade students at the distinguished level in mathematics: research and appraise real-world examples of situations by measuring and estimating time, liquid volume or masses of objects; organize and present real world data on a variety of graphs to justify solutions to problems based on data; communicate the use of area in the real-world and justify short cuts for finding area using addition, multiplication or tiling of non-rectilinear shapes; construct viable arguments and critique the reasoning of others in solving real-world problems involving perimeter and areas.</td>
<td>Third grade students at the above mastery level in mathematics: analyze real-world problems by using appropriate tools to measure and estimate time, liquid volume and masses of objects; check for reasonableness of solutions to problems using data from graphs and line plots; critique the correspondences between different approaches in finding area; communicate ways to compare and contrast perimeters and areas of real-world plane figures.</td>
<td>Third grade students at the mastery level in mathematics: solve problems using measuring and estimating of liquid volume, object mass, and intervals of time to the nearest minute; create and use graphs to solve one- and two-step problems comparing data; measure objects to the nearest 1/8 or 1/4 inch and create a line plot; explain how area is determined in more than one way and area’s relationship to addition and multiplication; find perimeters of real-world plane figures specifying linear units and create rectangles with the same area and different perimeters and vice versa.</td>
<td>Third grade students at the partial mastery level in mathematics: tell time to the nearest five minute interval and choose appropriate tools to measure liquid volume and masses of objects; use data from graphs and line plots to solve one-step problems; find the area of various shapes by counting unit squares and given the algorithm for multiplication, multiply to find area of rectangles; find perimeter of plane figures without all sides labeled.</td>
<td>Third grade students at the novice level in mathematics: tell time in fifteen minute intervals; measure liquid volume and masses of objects; find and compare data on graphs and line plots; count unit squares to find area of rectilinear shapes; find perimeter when given all side lengths.</td>
</tr>
</tbody>
</table>
Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.

M.3.MD.1 tell and write time to the nearest minute, measure time intervals in minutes and solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram. (CCSS Math.3.MD.1)
- Compare equivalent units of time using hours and minutes.
  - Examples: 60 minutes = one hour, 30 minutes = one half of an hour
- Recognize key vocabulary and/or phrases associated with time.
  - Examples: Quarter ‘til = 15 minutes before; half past the hour = 30 minutes after the hour
- Compare the lengths of time to complete everyday activities
  - Examples: Brushing your teeth = about 2 minutes; riding the bus = about 20 minutes.
- Tell and write time in hours and half-hours using analog and digital clocks.
- Recognize hour, minute, and second hands on an analog clock.

M.3.MD.2 measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg) and liters (l) (Excludes compound units such as cm³ and finding the geometric volume of a container) and subtract, multiply or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem. (Excludes multiplicative comparison problems - problems involving notions of “times as much”). (CCSS Math.3.MD.2)
- Define liquid volume, mass, grams, kilograms, and liters.
- Recognize how the standard units of measure compare to one another.
- Identify key terms for word problems.
  - Examples: Difference, altogether, in all, between
- Express the length of an object as a whole number of length units by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps.
- Recall basic addition, subtraction, multiplication, and division facts.
- Describe measurable attributes of objects such as length or weight. Describe several measurable attributes of a single object.

Represent and interpret data.

M.3.MD.3 draw a scaled picture graph and a scaled bar graph to represent a data set with several categories and solve one- and two-step “how many more” and “how many less” problems using information presented in scaled bar graphs. For example, draw a bar graph in which each square in the bar graph might represent 5 pets. (CCSS Math.3.MD.3)
- Define picture graph, bar graph, and data.
- Interpret the data to solve problems.
- Identify the parts of a graph (x-axis, y-axis, title, key, equal intervals, labels).
- Locate the data on a picture graph and a bar graph.
- Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another.
- Directly compare two objects, with a measurable attribute in common, to see which object has “more of” or “less of” the attribute, and describe the difference.

M.3.MD.4 generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch and show the data by making a line plot, where the horizontal scale is marked off in appropriate units—whole numbers, halves or quarters. (CCSS Math.3.MD.4)
- Define line plot.
- Identify the parts of a line plot.
- Measure objects to the nearest inch.
- Identify one-inch units on a ruler starting with 0.
- Express the length of an object as a whole number of length units by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps.
- Directly compare two objects, with a measurable attribute in common, to see which object has “more of” or “less of” the attribute, and describe the difference.

Geometric measurement: understand concepts of area and relate area to multiplication and to addition.

M.3.MD.5 recognize area as an attribute of plane figures and understand concepts of area measurement
a. a square with side length 1 unit, called “a unit square,” is said to have “one square unit” of area and can be used to measure area, (CCSS Math.3.MD.5)
- Define plane figures.
- Differentiate between closed and open figures.
- Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g., color, orientation, overall size); build and draw shapes to possess defining attributes.
- Identify shapes as two-dimensional (i.e., lying in a plane, “flat”).
- Correctly name shapes regardless of their orientations or overall size.
a. A square with side length 1 unit called “a unit square,” is said to have “one square unit” of area and can be used to measure area.
   i. Define length.
   ii. Recognize that units of measure must be equal.
   iii. Express the length of an object as a whole number of length units by laying multiple copies of a shorter object (the length unit) end to end.
   iv. Recognize that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps.

b. A plane figure which can be covered without gaps or overlaps by n unit squares is said to have an area of n square units.
   i. Define area.
   ii. Recognize that “n square units” is a variable.
   iii. Recognize that unit squares are equal.

M.3.MD.6 measure areas by counting unit squares (square cm, square m, square in, square ft and improvised units). (CCSS Math.3.MD.6)
   • Recognize that unit squares are equal.
   • Define the units of measurement (cm, m, in, ft).
   • Express the length of an object as a whole number of length units by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps.

M.3.MD.7 relate area to the operations of multiplication and addition.
   a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.
   b. Multiply side lengths to find areas of rectangles with whole number side lengths in the context of solving real-world and mathematical problems and represent whole-number products as rectangular areas in mathematical reasoning.
   c. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and b + c is the sum of a × b and a × c and use area models to represent the distributive property in mathematical reasoning.
   d. Recognize area as additive and find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real-world problems.

(CCSS Math.3.MD.7)
   • Recognize arrays as multiplication or repeated addition.
   • Recall basic addition and multiplication facts.
   • Build and draw shapes to possess defining attributes.
   • Compose simple shapes to form larger shapes.
   a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.
      i. Recognize arrays as multiplication or repeated addition.
      ii. Identify units of measure as equal units.
      iii. Build and draw shapes to possess defining attributes.
      iv. Compose simple shapes to form larger shapes.
   b. Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real-world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.
      i. Recall basic multiplication facts.
      ii. Express the length of an object as a whole number of length units by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps.
      iii. Recognize multiplication as repeated addition.
      iv. Add within 100.
   c. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and b + c is the sum of a × b and a × c. Use area models to represent the distributive property in mathematical reasoning.
      i. Define distributive property.
      ii. Label pre-made arrays.
      iii. Partition a rectangle into rows and columns of same-size squares, and count to find the total number of them.
      iv. Add within 100.
   d. Recognize area as additive. Find areas of rectilinear figures by decomposing them into nonoverlapping rectangles and adding the areas of the nonoverlapping parts, applying this technique to solve real-world problems.
      i. Label pre-made arrays.
      ii. Partition a rectangle into rows and columns of same-size squares, and count to find the total number of them.
      iii. Recall basic addition and multiplication facts.
      iv. Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles).
      v. Identify a rectangle.
Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.

M.3.MD.8 solve real-world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters. (CCSS Math.3.MD.8)

- Define perimeter.
- Recall the formula for perimeter \( P = L + L + W + W \) or \( P = 2L + 2W \).
- Recall basic addition and multiplication facts.
- Build and draw shapes to possess defining attributes.
- Express the length of an object as a whole number of length units by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps.
- Describe measurable attributes of objects such as length or weight.

### Geometry

<table>
<thead>
<tr>
<th>Performance Descriptors</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Distinguished</strong></td>
</tr>
<tr>
<td>Third grade students at the distinguished level in mathematics: analyze the relationship among shapes sharing attributes, justify the placement of shapes into various groups, and demonstrate equal parts of shapes as fractions of the whole.</td>
</tr>
<tr>
<td><strong>Above Mastery</strong></td>
</tr>
<tr>
<td>Third grade students at the above mastery level in mathematics: produce a display comparing and contrasting shapes by attributes and identify equal parts of shapes as fractions of the shape.</td>
</tr>
<tr>
<td><strong>Mastery</strong></td>
</tr>
<tr>
<td>Third grade students at the mastery level in mathematics: classify and describe shapes by attributes showing that some groups overlap; model equal parts of various shapes to express the part as a fraction of the whole.</td>
</tr>
<tr>
<td><strong>Partial Mastery</strong></td>
</tr>
<tr>
<td>Third grade students at the partial mastery level in mathematics: classify shapes by one attribute and show fractional parts.</td>
</tr>
<tr>
<td><strong>Novice</strong></td>
</tr>
<tr>
<td>Third grade students at the novice level in mathematics: identify quadrilaterals and sort concrete shapes by number of sides.</td>
</tr>
</tbody>
</table>

### Reason with Shapes and Their Attributes

M.3.G.1 understand that shapes in different categories (e.g., rhombuses, rectangles and others) may share attributes (e.g., having four sides), that the shared attributes can define a larger category (e.g. quadrilaterals), recognize rhombuses, rectangles and squares as examples of quadrilaterals and draw examples of quadrilaterals that do not belong to any of these subcategories.(CCSS Math.3.G.1)

- Recall the vocabulary of shapes (labels, sides, faces, vertices, etc.).
- Recognize and draw shapes having specified attributes such as a given number of angles.
- Build and draw shapes to possess defining attributes.
- Sort shapes into categories.

M.3.G.2 partition shapes into parts with equal areas and express the area of each part as a unit fraction of the whole. For example, partition a shape into 4 parts with equal area, and describe the area of each part as \( \frac{1}{4} \) of the area of the shape. (CCSS Math.3.G.2)

- Recognize a fraction as part of a whole.
- Decompose a large pre-made shape using smaller shapes.
- Compose a large pre-made shape using smaller shapes.
- Partition a rectangle into rows and columns of same-size squares, and count to find the total number of them.
- Partition circles and rectangles into two, three, or four equal shares; describe the shares using the words halves, thirds, half of, a third of, etc.; and describe the whole as two halves, three thirds, or four fourths.
Content Emphases by Cluster — Grade 4*

Not all of the content in a given grade is emphasized equally in the standards. Some clusters require greater emphasis than the others based on the depth of the ideas, the time that they take to master, and/or their importance to future mathematics or the demands of college and career readiness. In addition, an intense focus on the most critical material at each grade allows depth in learning, which is carried out through the Standards for Mathematical Practice.

To say that some things have greater emphasis is not to say that anything in the standards can safely be neglected in instruction. Neglecting material will leave gaps in student skill and understanding and may leave students unprepared for the challenges of a later grade. The following table identifies the Major Clusters, Additional Clusters, and Supporting Clusters for this grade.

Key: ■ Major Clusters; □ Supporting Clusters; ◊ Additional Clusters

Operations and Algebraic Thinking
■ Use the four operations with whole numbers to solve problems.
□ Gain familiarity with factors and multiples.
□ Generate and analyze patterns.

Number and Operations in Base Ten
■ Generalize place value understanding for multi-digit whole numbers.
□ Use place value understanding and properties of operations to perform multi-digit arithmetic.

Number and Operations--Fractions
■ Extend understanding of fraction equivalence and ordering.
□ Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.
□ Understand decimal notation for fractions, and compare decimal fractions.

Measurement and Data
□ Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.
□ Represent and interpret data.
□ Geometric measurement: understand concepts of angle and measure angles.

Geometry
□ Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

*Emphases are given at the cluster level. Refer to the Common Core State Standards for Mathematics for the specific standards that fall within each cluster.
Grade Focus Areas in Support of Rich Instruction and Expectations of Fluency and Conceptual Understanding

<table>
<thead>
<tr>
<th>Grade</th>
<th>Focus Areas</th>
</tr>
</thead>
<tbody>
<tr>
<td>K-2</td>
<td>Addition and subtraction, measurement using whole number quantities</td>
</tr>
<tr>
<td>3-5</td>
<td>Multiplication and division of whole numbers and fractions</td>
</tr>
</tbody>
</table>

In Grade 4, instructional time should focus on three critical areas: (1) developing understanding and fluency with multi-digit multiplication and developing understanding of dividing to find quotients involving multi-digit dividends; (2) developing an understanding of fraction equivalence, addition and subtraction of fractions with like denominators and multiplication of fractions by whole numbers; (3) understanding that geometric figures can be analyzed and classified based on their properties, such as having parallel sides, perpendicular sides, particular angle measures and symmetry.

1. Students generalize their understanding of place value to 1,000,000, understanding the relative sizes of numbers in each place. They apply their understanding of models for multiplication (equal-sized groups, arrays, area models), place value and properties of operations, in particular the distributive property, as they develop, discuss and use efficient, accurate and generalizable methods to compute products of multi-digit whole numbers. Depending on the numbers and the context, they select and accurately apply appropriate methods to estimate or mentally calculate products. They develop fluency with efficient procedures for multiplying whole numbers; understand and explain why the procedures work based on place value and properties of operations and use them to solve problems. Students apply their understanding of models for division, place value, properties of operations and the relationship of division to multiplication as they develop, discuss and use efficient, accurate and generalizable procedures to find quotients involving multi-digit dividends. They select and accurately apply appropriate methods to estimate and mentally calculate quotients and interpret remainders based upon the context.

2. Students develop understanding of fraction equivalence and operations with fractions. They recognize that two different fractions can be equal (e.g., 15/9 = 5/3), and they develop methods for generating and recognizing equivalent fractions. Students extend previous understandings about how fractions are built from unit fractions, composing fractions from unit fractions, decomposing fractions into unit fractions and using the meaning of fractions and the meaning of multiplication to multiply a fraction by a whole number.

3. Students describe, analyze, compare and classify two-dimensional shapes. Through building, drawing and analyzing two-dimensional shapes, students deepen their understanding of properties of two-dimensional objects and the use of them to solve problems involving symmetry.

Mathematical Practices
1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

<table>
<thead>
<tr>
<th>Grade</th>
<th>Standard</th>
<th>Required Fluencies</th>
</tr>
</thead>
<tbody>
<tr>
<td>K</td>
<td>K.OA.5</td>
<td>Add/subtract within 5</td>
</tr>
<tr>
<td>1</td>
<td>1.OA.6</td>
<td>Add/subtract within 10</td>
</tr>
<tr>
<td>2</td>
<td>2.OA.2</td>
<td>Add/subtract within 20 (know single-digit sums from memory)</td>
</tr>
<tr>
<td></td>
<td>2.NBT.5</td>
<td>Add/subtract within 100</td>
</tr>
<tr>
<td>3</td>
<td>3.OA.7</td>
<td>Multiply/divide within 100 (know single-digit products from memory)</td>
</tr>
<tr>
<td></td>
<td>3.NBT.2</td>
<td>Add/subtract within 1,000</td>
</tr>
<tr>
<td>4</td>
<td>4.NBT.4</td>
<td>Add/subtract within 1,000,000</td>
</tr>
</tbody>
</table>
## Operations & Algebraic Thinking (OA)

<table>
<thead>
<tr>
<th>Performance Descriptors</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Distinguished</strong></td>
</tr>
<tr>
<td>Fourth grade students at the distinguished level in mathematics: justify, communicate, defend conclusions, and respond to the arguments of others; analyze and reflect on relationships among factors and multiples and draw conclusions; analyze relationships in a given pattern and discover a rule.</td>
</tr>
</tbody>
</table>

| **Above Mastery**         |
| Fourth grade students at the above mastery level in mathematics: use various strategies to solve and check multi-step word problems and precisely communicate procedures used; identify significance of the factor/multiple relationships; justify and communicate a pattern or given rule. |

| **Mastery**               |
| Fourth grade students at the mastery level in mathematics: distinguish between multiplicative and additive reasoning, apply the four operations with whole numbers to solve multi-step word problems, represent problems with equations containing unknowns, and evaluate the reasonableness of the results; find and make connections between factors/multiples and prime/composite numbers; generate and/or discern a pattern or structure when given a rule. |

| **Partial Mastery**       |
| Fourth grade students at the partial mastery level in mathematics: distinguish between multiplicative and additive reasoning and apply the four operations with whole numbers to solve multi-step word problems; given a list of multiples of a number, determine the number and given a number, determine some of its multiples and factors; extend or complete a pattern or structure. |

| **Novice**                |
| Fourth grade students at the novice level in mathematics: apply the four operations with whole numbers to solve simple one-step word problems; state multiplication facts and use concrete objects or pictures to model factors and multiples; identify missing elements in a given pattern. |

### Use the four operations with whole numbers to solve problems.

**M.4.OA.1** interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 \times 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5 and represent verbal statements of multiplicative comparisons as multiplication equations. (CCSS Math.4.OA.1)

- Use arrays to show equal groups in multiplication.
- Recall basic multiplication facts.
- Interpret the products of whole numbers.
- Demonstrate computational fluency, including quick recall of addition and subtraction facts.
- Recognize multiplication as repeated addition.

**M.4.OA.2** multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem and distinguishing multiplicative comparison from additive comparison. (CCSS Math.4.OA.2)

- Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.
- Recognize key terms to solve word problems.
- Examples: in all, how much, how many, in each
- Apply properties of operations as strategies to add.
- Recall basic multiplication facts.
- Demonstrate computational fluency, including quick recall of addition and subtraction facts.

**M.4.OA.3** solve multi-step word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted, represent these problems using equations with a letter standing for the unknown quantity and assess the reasonableness of answers using mental computation and estimation strategies including rounding. (CCSS Math.4.OA.3)

- Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.
- Solve single-step word problems.
- Recognize key terms to solve word problems.
- Examples: in all, how much, how many, in each
- Solve division problems without remainders.
- Recall basic addition, subtraction, and multiplication facts.

### Gain familiarity with factors and multiples

**M.4.OA.4** find all factor pairs for a whole number in the range 1–100, recognize that a whole number is a multiple of each of its factors, determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number and determine whether a given whole number in the range 1–100 is prime or composite. (CCSS Math.4.OA.4)

- Define factors, prime number, and composite number.
- Apply properties of operations as strategies to multiply and divide.
- Identify all factor pairs for a whole number in the range 1-20.
- Name the first ten multiples of each one-digit natural number.
- Recall basic multiplication facts.
- Count within 1000; skip-count by 5s, 10s, and 100s.
Generate and analyze patterns.

M.4.OA.5 generate a number or shape pattern that follows a given rule and identify apparent features of the pattern that were not explicit in the rule itself. For example, given the rule “Add 3” and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way. (CCSS Math.4.OA.5)

- Identify arithmetic patterns, including patterns in the addition table or multiplication table and explain them using properties of operations.
- Recognize arithmetic patterns (including geometric patterns or patterns in the addition table or multiplication table).

Examples: Continue a geometric pattern $\bigtriangleup \bigcirc \bigtriangleup \bigcirc \ldots$ by drawing the next three shapes.

Sample Answer: $\bigtriangleup \bigcirc \bigtriangleup$

- Construct repeating and growing patterns with a variety of representations.
- Continue an existing pattern.
- Identify arithmetic patterns.
- Demonstrate computational fluency, including quick recall, of addition multiplication facts.

Number & Operations in Base Ten

<table>
<thead>
<tr>
<th>Performance Descriptors</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Distinguished</th>
<th>Above Mastery</th>
<th>Mastery</th>
<th>Partial Mastery</th>
<th>Novice</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fourth grade students at the distinguished level in mathematics: connect place value to other mathematical concepts, including time, money, and measurement; communicate reasoning, analyze situations, and justify solutions.</td>
<td>Fourth grade students at the above mastery level in mathematics: explain place value relationships and knowledge of rounding and justify reasoning; evaluate the reasonableness of intermediate results while performing multi-digit arithmetic.</td>
<td>Fourth grade students at the mastery level in mathematics: demonstrate understanding of place value and rounding of whole numbers; illustrate and explain place value and apply properties of operations to perform multi-digit arithmetic.</td>
<td>Fourth grade students at the partial mastery level in mathematics: make sense of place value relationships for multi-digit whole numbers; apply place value understanding and properties of operations to perform simple multi-digit arithmetic.</td>
<td>Fourth grade students at the novice level in mathematics: use concrete objects or pictures to help conceptualize and understand the base ten system; procedurally perform addition and subtraction problems and know basic multiplication facts.</td>
</tr>
</tbody>
</table>

Generalize place value understanding for multi-digit whole numbers.

M.4.NBT.1 recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. For example, recognize that $700 \div 70 = 10$ by applying concepts of place value and division. (CCSS Math.4.NBT.1)

- Use place value understanding to round whole numbers to the nearest 10 or 100.
- Add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.
- Multiply one-digit whole numbers by multiples of 10 in the range 10 - 90 (e.g., $9 \times 80, 5 \times 60$) using strategies based on place value and properties of operations.
- Recall basic multiplication facts.
- Recall that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones.
- Recognize that the numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).
- Recognize that 100 can be thought of as a bundle of ten tens, called a “hundred.”

M.4.NBT.2 read and write multi-digit whole numbers using base-ten numerals, number names and expanded form and compare two multi-digit numbers based on meanings of the digits in each place, using $>$, $=$ and $<$ symbols to record the results of comparisons. (CCSS Math.4.NBT.2)

- Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits using $>$, $=$, and $<$ symbols to record the results of comparisons.
- Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.
- Convert a number written in expanded notation to standard form.

M.4.NBT.3 use place value understanding to round multi-digit whole numbers to any place. (CCSS Math.4.NBT.3)

- Use place value understanding to round whole numbers to the nearest 10 or 100.
- Model rounding whole numbers to the nearest 100.
- Round whole numbers from 100 to 999 using whole numbers from 10 to 99.
- Model rounding whole numbers to the nearest 10.
- Round whole numbers from 10 to 99 using whole numbers from 1 to 9.
- Round whole numbers from 1 to 9 and model to show proficiency.
Use place value understanding and properties of operations to perform multi-digit arithmetic.

M.4.NBT.4 fluently add and subtract multi-digit whole numbers using the standard algorithm. (CCSS Math.4.NBT.4)
- Add and subtract within 1000.
- Apply signs +, -, and = to actions of joining and separating sets.
- Add and subtract single-digit numbers.
- Recall basic addition and subtraction facts.

M.4.NBT.5 multiply a whole number of up to four digits by a one-digit whole number, multiply two two-digit numbers, using strategies based on place value and the properties of operations and illustrate and explain the calculation by using equations, rectangular arrays and/or area models. (CCSS Math.4.NBT.5)
- Multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations.
- Multiply single-digit numbers.
- Recall basic multiplication facts.
- Apply concepts of multiplication through the use of manipulatives, number stories, skipcounting arrays, area of a rectangle, or repeated addition.

M.4.NBT.6 find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations and/or the relationship between multiplication and division and illustrate and explain the calculation by using equations, rectangular arrays and/or area models. (CCSS Math.4.NBT.6)
- Divide within 100, using strategies such as the relationship between multiplication and division (e.g. knowing that 8 x 5 = 40, one knows 40 ÷ 5 = 8).
- Divide within 100, using strategies such as properties of operations.
- Multiply within 100, using strategies such as properties of operations.
- Name the first 10 multiples of each one-digit natural number.
- Example: 7, 14, 21, 28, 35, 42, 49, 56, 63, 70
- Recall basic addition, subtraction, and multiplication facts.

Number & Operations - Fractions

<table>
<thead>
<tr>
<th>Performance Descriptors</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Distinguished</strong></td>
</tr>
<tr>
<td>Fourth grade students at the distinguished level in mathematics:</td>
</tr>
<tr>
<td>use and apply knowledge of equivalent fractions as a strategy to solve real-world problems in new situations;</td>
</tr>
<tr>
<td>extend knowledge of fractions by applying mathematics to solve problems arising in everyday life, society and the workplace; justify correspondences between fraction and decimal comparisons.</td>
</tr>
</tbody>
</table>

Extend understanding of fraction equivalence and ordering.

M.4.NF.1 explain why a fraction a/b is equivalent to a fraction (n × a)/(n × b) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size and use this principle to recognize and generate equivalent fractions. (CCSS Math.4.NF.1)
- Define fraction, numerator and denominator.
- Recognize fraction 1/b as the quantity formed by 1 part when a whole is partitioned into b equal parts.
- Recognize fractions as numerals that may represent division problems.
- Label numerator, denominator, and fraction bar.
- Identify parts of a whole with two, three, or four equal parts.
- Recognize that equal shares of identical wholes need not have the same shape.
- Distinguish between equal and non-equal parts.
M.4.NF.2 compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as \( \frac{1}{2} \), recognize that comparisons are valid only when the two fractions refer to the same whole and record the results of comparisons with symbols \( >, = \) or \( < \), and justify the conclusions, e.g., by using a visual fraction model. (CCSS Math.4.NF.2)

- Identify fraction \( \frac{1}{b} \) as the quantity formed by \( 1 \) part when a whole is partitioned into \( b \) equal parts; understand a fraction \( \frac{a}{b} \) as the quantity formed by \( a \) parts of size \( \frac{1}{b} \).
- Identify a fraction as a number on the number line; represent fractions on a number line diagram.
- Recognize a fraction as a number on the number line.
- Represent fractions on a number line diagram.
- Recognize fractions as numerals that may represent division problems.
- Label numerator, denominator, and fraction bar.
- Identify parts of a whole with two, three, or four equal parts.
- Distinguish between equal and non-equal parts.

Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

M.4.NF.3 understand a fraction \( \frac{a}{b} \) with \( a > 1 \) as a sum of fractions \( \frac{1}{b} \)

a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.
- Identify addition and subtraction of fractions as joining and separating parts referring to the same whole.
- Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation and justify decompositions, e.g., by using a visual fraction model.
- Examples: \( \frac{3}{8} = \frac{1}{8} + \frac{1}{8} + \frac{1}{8}; \frac{3}{8} = \frac{1}{8} + \frac{2}{8}; \frac{2}{1} = \frac{1}{1} + \frac{1}{8} = \frac{8}{8} + \frac{8}{8} + \frac{1}{8} \).

b. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction and/or by using properties of operations and the relationship between addition and subtraction.
- Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction and/or by using properties of operations and the relationship between addition and subtraction.
- Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.

(CCSS Math.4.NF.3)

- Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.
- Identify two fractions as equivalent (equal) if they are the same size or the same point on a number line.
- Recognize and generate simple equivalent fractions. Explain why the fractions are equivalent, e.g., by using a visual fraction model.
- Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers.
- Label a fraction with multiple representations.
- Recognize pictorial representations of equivalent fractions.
- Recognize different interpretations of fractions, including parts of a set or a collection, points on a number line, numbers that lie between two consecutive whole numbers, and lengths of segments on a ruler.
- Recognize that a whole can be partitioned into differing equal parts (halves, fourths, eighths, etc.).
  a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.
    - Identify numerator and denominator.
    - Recall basic addition and subtraction facts.
  b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model.
    - Demonstrate an understanding of fractional parts.
    - Recall basic addition and subtraction facts.
  c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.
    - Define mixed numbers.
    - Recall basic addition and subtraction facts.
  d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.
    - Demonstrate an understanding of fractional parts.
    - Solve word problems using whole numbers.
    - Express parts of a whole as a fraction.
    - Write number sentences for word problems.
    - Identify key terms in word problems.
    - Recall basic addition and subtraction facts.

M.4.NF.4 apply and extend previous understandings of multiplication to multiply a fraction by a whole number

a. Understand a fraction \( \frac{a}{b} \) as a multiple of \( \frac{1}{b} \). (For example, use a visual fraction model to represent \( \frac{5}{4} \) as the product \( 5 \times \frac{1}{4} \), recording the conclusion by the equation \( \frac{5}{4} = 5 \times \frac{1}{4} \).
- Recall basic addition and subtraction facts.
- Define mixed numbers.
- Demonstrate an understanding of fractional parts.
- Solve basic word problems using whole numbers.
- Express parts of a whole as a fraction.
- Write number sentences for word problems.
- Identify key terms in word problems.
- Recall basic addition and subtraction facts.

b. Understand a multiple of \( \frac{a}{b} \) as a multiple of \( \frac{1}{b} \), and use this understanding to multiply a fraction by a whole number, [For example, use a visual fraction model to express \( \frac{3}{2} / (5/2) \) as \( 3 \times (1/5) \), recognizing this product as \( \frac{6}{5} \). In general, \( n \times (a/b) = (n \times a) / b \).
- Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. (For example, if each person at a party will eat \( 3/8 \) of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?)
- Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.
  a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.
  - Identify numerator and denominator.
  - Recall basic addition and subtraction facts.
  b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model.
    - Demonstrate an understanding of fractional parts.
    - Recall basic addition and subtraction facts.
  c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.
    - Define mixed numbers.
    - Recall basic addition and subtraction facts.
  d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.
    - Demonstrate an understanding of fractional parts.
    - Solve word problems using whole numbers.
    - Express parts of a whole as a fraction.
    - Write number sentences for word problems.
    - Identify key terms in word problems.
    - Recall basic addition and subtraction facts.
Guidance for West Virginia Schools and Districts

(CCSS Math.4.NF.4)
- Recognize fractions in their simplest forms.
- Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers.
- Demonstrate an understanding of fractional parts.
- Apply properties of operations as strategies to multiply and divide.
- Recall basic multiplication facts.
  a. Understand a fraction $a/b$ as a multiple of $1/b$
    ▫ Define multiple.
    ▫ Compare two fractions with the same numerator or the same denominator by reasoning about their size.
    ▫ Recognize that comparisons are valid only when the two fractions refer to the same whole.
    ▫ Record results of comparisons with the symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using a visual fraction model.
    ▫ Name the first ten multiples of each one-digit natural number.
    ▫ Recall basic multiplication facts.
  b. Understand a multiple of $a/b$ as a multiple of $1/b$, and use this understanding to multiply a fraction by a whole number.
    Example: Use a visual fraction model to express $3 \times (2/5)$ as $6 \times (1/5)$, recognizing this product as $6/5$. (In general, $n \times (a/b) = (nxa)/b$.)
    ▫ Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers.
    ▫ Solve simple fractions using multiplication strategies.
    ▫ Recognize equivalent forms of fractions.
  c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem.
    Example: If each person at a party will eat $3/8$ of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between which two whole numbers does your answer lie?
    ▫ Multiply proper fractions with common denominators 2-10.
    ▫ Solve word problems using whole numbers.
    ▫ Write number sentences for word problems.
    ▫ Identify key terms in word problems.
    ▫ Multiply and divide within 100.

Understand decimal notation for fractions, and compare decimal fractions.

M.4.NE.5 express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100. For example, express $3/10$ as $30/100$, and add $3/10 + 4/100 = 34/100$. (Students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators in general. But addition and subtraction with unlike denominators in general is not a requirement at this grade.) (CCSS Math.4.NF.5)
- Recognize equivalent forms of fractions and decimals.
- Demonstrate equivalent fractions using concrete objects or pictorial representation.
- Recognize pictorial representations of equivalent fractions and decimals in tenths and hundredths.
  Example: $0.60 = 0.6$
- Identify place value of decimals to the tenths and hundredths.
- Use place value understanding to round whole numbers to the nearest 10 or 100.

M.4.NE.6 use decimal notation for fractions with denominators 10 or 100. For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram. (CCSS Math.4.NF.6)
- Define tenths, hundredths, decimal notation.
- Recognize equivalent forms of fractions and decimals.
- Recognize that endpoints locate $a/b$ on a number line.
- Identify place value of decimals to the tenths and hundredths.
- Label fraction parts.
  Examples: numerator, denominator, fraction bar
- Use place value understanding to round whole numbers to the nearest 10 or 100.

M.4.NE.7 compare two decimals to hundredths by reasoning about their size, recognize that comparisons are valid only when the two decimals refer to the same whole and record the results of comparisons with the symbols $>$, $=$, or $<$ and justify the conclusions, e.g., by using a visual model. (CCSS Math.4.NF.7)
- Compare two fractions with the same numerator or the same denominator by reasoning about their size.
- Recognize that comparisons are valid only when the two fractions refer to the same whole.
- Record the results of comparisons with the symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using a visual fraction model.
- Convert fractions to decimals.
- Compare two decimals to tenths.
- Compare whole numbers.
- Identify comparison symbols.
  Examples: $>$, $<$, and $=$
Measurement & Data

<table>
<thead>
<tr>
<th>Performance Descriptors</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Distinguished</strong></td>
</tr>
<tr>
<td>Fourth grade students at the distinguished level in mathematics:</td>
</tr>
<tr>
<td>analyze arguments and justify reasoning;</td>
</tr>
<tr>
<td>identify a real-world problem, design and conduct experiments involving measurements, record and display data, analyze, and communicate results; design a project demonstrating knowledge of angle concepts and measurements and present final product.</td>
</tr>
</tbody>
</table>

Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.

**M.4.MD.1** know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec, within a single system of measurement, express measurements in a larger unit in terms of a smaller unit, record measurement equivalents in a two column table, (For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in.) and generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36). (CCSS Math.4.MD.1)

- Define conversion.
- Define length, kilometers, meters and centimeters.
- Define weight, kilograms, grams, pounds, ounces, liters and milliliters.
- Define hour, minute, second.
- Measure and estimate liquid volumes and masses of objects using standard units of grams, kilograms, and liters.
- Identify standard units of measurement equivalents. Examples: 60 minutes equals 1 hour, 16 ounces equals 1 pound
- Match measurement units to abbreviations. Examples: kilometers (km), meters (m), centimeters (cm), kilograms (kg), grams (g), pounds (lb), ounces (oz), liters (l), milliliters (ml)

**M.4.MD.2** use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects and money, including problems involving simple fractions or decimals and problems that require expressing measurements given in a larger unit in terms of a smaller unit and represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale. (CCSS Math.4.MD.2)

- Define distance, time, elapsed time, volume, mass.
- Determine elapsed time to the day with calendars and to the hour with a clock.
- Express liquid volumes and masses of objects using standard units of grams, kilograms, and liters.
- Use addition, subtraction, multiplication and division to solve one- and two-step word problems.
- Recognize key terms to solve word problems.
- Recall basic facts for addition, subtraction, multiplication, and division.
- Identify monetary equivalents. Examples: four quarters equal one dollar, five one-dollar bills equals five dollars

**M.4.MD.3** apply the area and perimeter formulas for rectangles in real world and mathematical problems. (For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.) (CCSS Math.4.MD.3)

- Recall the formula for area (L X W).
- Recognize that unit squares are equal.
- Recall the formula for perimeter (P= L+L+W+W or P=2L + 2W).
- Recall basic addition and multiplication facts.
Represent and interpret data.

M.4.MD.4 Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8) and solve problems involving addition and subtraction of fractions by using information presented in line plots (For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection). (CCSS Math.4.MD.4)

• Display data by making a line plot where the horizontal scale is marked off in appropriate units – whole numbers, halves, or quarters.
• Interpret data using graphs including bar, line, and circle graphs, and Venn diagrams.
• Identify the parts of a line plot.
• Recognize a line plot.
• Draw a scaled picture graph and a scaled bar graph to represent a data set.

Geometric measurement: understand concepts of angle and measure angles.

M.4.MD.5 Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement:

a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle and an angle that turns through 1/360 of a circle is called a “one-degree angle,” and can be used to measure angles,

b. An angle that turns through n one-degree angles is said to have an angle measure of n degrees. (CCSS Math.4.MD.5)

• Define degree, angle, ray, and vertices.
• Recognize and draw shapes having specified attributes such as a given number of angles or a given number of equal faces.
• Estimate angle measures using 45°, 90°, 180°, 270°, or 360°.
• Identify angle, ray, and vertices.
• Draw shapes to possess defining attributes.

a. An angle is measured with reference to a circle with its center at the common endpoint of the rays by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a “one-degree angle” and can be used to measure angles.

• Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole.
  - Example: Partition a shape into 4 parts with equal area, and describe the area of each part as ¼ of the area of the shape.
• Partition circles and rectangles into two, three, or four equal shares; describe the shares using the words halves, thirds, half of, a third of, etc.; and describe the whole as two halves, three thirds, or four fourths.
• Recognize that equal shares of identical wholes need not have the same shape.
• Demonstrate equivalent fractions using concrete objects or pictorial representations.

b. An angle that turns through n one-degree angles is said to have an angle measure of n degrees.

• Define center, radius, and diameter of a circle.
• Identify real-world examples of radius and diameter.
  - Examples: bicycle wheel, pizza, pie
• Identify intervals of 1° between 0 and 5 on a protractor.
• Skip count by fives and tens on a protractor.

M.4.MD.6 Measure angles in whole-number degrees using a protractor and sketch angles of specified measure. (CCSS Math.4.MD.6)

• Define symmetry.
• Model using a protractor to draw angles.
• Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen.
• Measure the length of an object by selecting and using appropriate tools such as a ruler.
• Measure length using standard and non-standard units of measurement.
• Plot points on grids, graphs, and maps using coordinates.

M.4.MD.7 Recognize angle measure as additive, when an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts and solve addition and subtraction problems to find unknown angles on a diagram in real-world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure. (CCSS Math.4.MD.7)

• Identify straight angles.
• Recognize angle measures such as 45°, 90°, 180°, 270°, 300°.
• Recall basic addition and subtraction facts.
• Skip count by fives and tens.
Geometry

<table>
<thead>
<tr>
<th>Performance Descriptors</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Distinguished</strong></td>
</tr>
<tr>
<td>Fourth grade students at the distinguished level in mathematics: justify conclusions and respond to the arguments of others.</td>
</tr>
</tbody>
</table>

**Draw and Identify Lines and Angles and Classify Shapes by Properties of Their Lines and Angles**

M.4.G.1 draw points, lines, line segments, rays, angles (right, acute, obtuse) and perpendicular and parallel lines and identify these in two-dimensional figures. (CCSS Math.4.G.1)
- Define points, lines, line segments, rays, right angle, acute angle, obtuse angle, perpendicular lines, and parallel lines.
- Define two-dimensional figure.
- Recognize one-dimensional points, lines, and line segments.
- Model shapes in the world by building shapes from components.

M.4.G.2 classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines or the presence or absence of angles of a specified size, recognize right triangles as a category and identify right triangles. (CCSS Math.4.G.2)
- Define right angle.
- Recognize that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals).
- Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.
- Recognize and draw shapes having specified attributes such as a given number of angles or a given number of equal faces.
- Identify triangles.

M.4.G.3 recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts, identify line-symmetric figures and draw lines of symmetry. (CCSS Math.4.G.3)
- Identify line symmetric figures.
- Draw lines of symmetry on a one-dimensional figure.
- Recognize lines of symmetry on a one-dimensional figure.
Mathematics (M) Grade 5

Content Emphases by Cluster--Grade 5*

Not all of the content in a given grade is emphasized equally in the standards. Some clusters require greater emphasis than the others based on the depth of the ideas, the time that they take to master, and/or their importance to future mathematics or the demands of college and career readiness. In addition, an intense focus on the most critical material at each grade allows depth in learning, which is carried out through the Standards for Mathematical Practice.

To say that some things have greater emphasis is not to say that anything in the standards can safely be neglected in instruction. Neglecting material will leave gaps in student skill and understanding and may leave students unprepared for the challenges of a later grade. The following table identifies the Major Clusters, Additional Clusters, and Supporting Clusters for this grade.

Key: ■ Major Clusters; □ Supporting Clusters; □ Additional Clusters

Operations and Algebraic Thinking
■ Write and interpret numerical expressions.
■ Analyze patterns and relationships.

Number and Operations in Base Ten
■ Understand the place value system.
■ Perform operations with multi-digit whole numbers and with decimals to hundredths.

Number and Operations—Fractions
■ Use equivalent fractions as a strategy to add and subtract fractions.
■ Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

Measurement and Data
■ Convert like measurement units within a given measurement system.
■ Represent and interpret data.
■ Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.

Geometry
□ Graph points on the coordinate plane to solve real-world and mathematical problems.
□ Classify two-dimensional figures into categories based on their properties.

*Emphases are given at the cluster level. Refer to the Common Core State Standards for Mathematics for the specific standards that fall within each cluster.
<table>
<thead>
<tr>
<th>Grade</th>
<th>Focus Areas in Support of Rich Instruction and Expectations of Fluency and Conceptual Understanding</th>
</tr>
</thead>
<tbody>
<tr>
<td>K-2</td>
<td>Addition and subtraction, measurement using whole number quantities</td>
</tr>
<tr>
<td>3-5</td>
<td>Multiplication and division of whole numbers and fractions</td>
</tr>
</tbody>
</table>

In Grade 5, instructional time should focus on three critical areas: (1) developing fluency with addition and subtraction of fractions and developing understanding of the multiplication of fractions and of division of fractions in limited cases (unit fractions divided by whole numbers and whole numbers divided by unit fractions); (2) extending division to 2-digit divisors, integrating decimal fractions into the place value system and developing understanding of operations with decimals to hundredths and developing fluency with whole number and decimal operations; and (3) developing understanding of volume.

1. Students apply their understanding of fractions and fraction models to represent the addition and subtraction of fractions with unlike denominators as equivalent calculations with like denominators. They develop fluency in calculating sums and differences of fractions, and make reasonable estimates of them. Students also use the meaning of fractions, of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for multiplying and dividing fractions make sense. (Note: this is limited to the case of dividing unit fractions by whole numbers and whole numbers by unit fractions.)

2. Students develop understanding of why division procedures work based on the meaning of base-ten numerals and properties of operations. They finalize fluency with multi-digit addition, subtraction, multiplication and division. They apply their understandings of models for decimals, decimal notation and properties of operations to add and subtract decimals to hundredths. They develop fluency in these computations and make reasonable estimates of their results. Students use the relationship between decimals and fractions, as well as the relationship between finite decimals and whole numbers (i.e., a finite decimal multiplied by an appropriate power of 10 is a whole number), to understand and explain why the procedures for multiplying and dividing finite decimals make sense. They compute products and quotients of decimals to hundredths efficiently and accurately.

3. Students recognize volume as an attribute of three-dimensional space. They understand that volume can be measured by finding the total number of same-size units of volume required to fill the space without gaps or overlaps. They understand that a 1-unit by 1-unit by 1-unit cube is the standard unit for measuring volume. They select appropriate units, strategies, and tools for solving problems that involve estimating and measuring volume. They decompose three-dimensional shapes and find volumes of right rectangular prisms by viewing them as decomposed into layers of arrays of cubes. They measure necessary attributes of shapes in order to determine volumes to solve real world and mathematical problems.

**Mathematical Practices**

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

<table>
<thead>
<tr>
<th>Grade</th>
<th>Standard</th>
<th>Required Fluencies</th>
</tr>
</thead>
<tbody>
<tr>
<td>K</td>
<td>K.OA.5</td>
<td>Add/subtract within 5</td>
</tr>
<tr>
<td>1</td>
<td>1.OA.6</td>
<td>Add/subtract within 10</td>
</tr>
<tr>
<td>2</td>
<td>2.OA.2</td>
<td>Add/subtract within 20 (know single-digit sums from memory)</td>
</tr>
<tr>
<td></td>
<td>2.NBT.5</td>
<td>Add/subtract within 100</td>
</tr>
<tr>
<td>3</td>
<td>3.OA.7</td>
<td>Multiply/divide within 100 (know single-digit products from memory)</td>
</tr>
<tr>
<td></td>
<td>3.NBT.2</td>
<td>Add/subtract within 1,000</td>
</tr>
<tr>
<td>4</td>
<td>4.NBT.4</td>
<td>Add/subtract within 1,000,000</td>
</tr>
<tr>
<td>5</td>
<td>5.NBT.5</td>
<td>Multi-digit multiplication</td>
</tr>
</tbody>
</table>
### Operations & Algebraic Thinking (OA)

#### Performance Descriptors

<table>
<thead>
<tr>
<th>Distinguished</th>
<th>Above Mastery</th>
<th>Mastery</th>
<th>Partial Mastery</th>
<th>Novice</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fifth grade students at the distinguished level in mathematics:</td>
<td>Fifth grade students at the above mastery level in mathematics:</td>
<td>Fifth grade students at the mastery level in mathematics:</td>
<td>Fifth grade students at the partial mastery level in mathematics:</td>
<td>Fifth grade students at the novice level in mathematics:</td>
</tr>
<tr>
<td>represent simple real-world situations with numerical expressions;</td>
<td>represent simple real-world situations with numerical expressions;</td>
<td>evaluate expressions, interpret the meaning of more complex expressions without evaluating them;</td>
<td>translate simple verbal phrases into numerical expressions;</td>
<td>evaluate given expressions;</td>
</tr>
<tr>
<td>create two or more rules and analyze the described patterns.</td>
<td>analyze and justify expressions to determine equivalence;</td>
<td>extend numerical patterns, make predictions and draw conclusions based on the patterns.</td>
<td>identify the relationship between terms in a given pattern and recognize the relationship between the ordered pairs and the graph of the terms of the pattern.</td>
<td>determine the rule, the input or output for a numerical pattern and write and graph ordered pairs.</td>
</tr>
</tbody>
</table>

#### Write and interpret numerical expressions.

M.5.OA.1 Use parentheses, brackets or braces in numerical expressions and evaluate expressions with these symbols. (CCSS Math.5.OA.1)
- Define parentheses, braces, and brackets.
- Define numerical expression.
- Recognize expressions.
- Apply properties of operations as strategies to add and subtract.
- Recall properties of operations as strategies to add and subtract.
- Represent addition and subtraction with objects, mental images, drawings, expressions, or equations.

M.5.OA.2 Write simple expressions that record calculations with numbers and interpret numerical expressions without evaluating them. For example, express the calculation “add 8 and 7, then multiply by 2” as 2 × (8 + 7). Recognize that 3 × (18932 + 921) is three times as large as 18932 + 921, without having to calculate the indicated sum or product. (CCSS Math.5.OA.2)
- Define simple expression.
- Recognize properties of addition and multiplication.
- Recall addition, subtraction, multiplication, division symbols.

#### Analyze patterns and relationships.

M.5.OA.3 Generate two numerical patterns using two given rules, identify apparent relationships between corresponding terms, form ordered pairs consisting of corresponding terms from the two patterns and graph the ordered pairs on a coordinate plane. For example, given the rule “Add 3” and the starting number 0 and given the rule “Add 6” and the starting number 0, generate terms in the resulting sequences and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so. (CCSS Math.5.OA.3)
- Construct repeating and growing patterns with a variety of representations.
- Continue an existing pattern.
- Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations.
- Recognize arithmetic patterns (including geometric patterns or patterns in the addition table or multiplication table).
  - Example: Continue a geometric pattern O Δ O Δ ___ ___ by drawing the next three shapes. Sample Answer: O Δ O
- Recall basic addition facts.
Number & Operations in Base Ten

<table>
<thead>
<tr>
<th>Performance Descriptors</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Distinguished</strong></td>
</tr>
<tr>
<td>Fifth grade students at the distinguished level in mathematics: justify the underlying structure and patterns of the place value system and extend the use of powers of ten to decimal numbers; justify strategies and procedures used to solve problems involving multi-digit whole numbers and decimals to hundredths.</td>
</tr>
</tbody>
</table>

**Understand the place value system.**

M.5.NBT.1 recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left. (CCSS Math.5.NBT.1)

- Use place value understanding to round whole numbers to the nearest 10 or 100.
- Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits using >, =, and < symbols to record the results of comparisons.
- Identify that the three digits of a three-digit number represent amounts of hundreds, tens, and ones.

M.5.NBT.2 explain patterns in the number of zeros of the product when multiplying a number by powers of 10, explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10 and use whole-number exponents to denote powers of 10. (CCSS Math.5.NBT.2)

- Define product, power of 10.
- Recognize decimal place value using visual representations.
- Recall multiplication and division facts of 10.
- Skip count forward and backward by 10.

M.5.NBT.3 read, write and compare decimals to thousandths

a. read and write decimals to thousandths using base-ten numerals, number names and expanded form, e.g., 347.392 = 3 × 100 + 4 × 10 + 7 × 1 + 3 × (1/10) + 9 × (1/100) + 2 × (1/1000),
b. compare two decimals to thousandths based on meanings of the digits in each place, using >, = and < symbols to record the results of comparisons.

(CCSS Math.5.NBT.3)

- Recognize decimals as parts of a whole.
- Compare whole numbers.
- Write whole numbers in words and expanded form.
- Read whole numbers.

a. Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., 347.392 = 3 × 100 + 4 × 10 + 7 × 1 + 3 × (1/10) + 9 × (1/100) + 2 × (1/1000),

- Define expanded notation and standard form.
- Convert a number written in expanded to standard form.
- Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.

b. Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.

- M.5.NBT.3a.1 Define hundredths and thousandths.
- M.5.NBT.3a.2 Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits using >, =, and < symbols to record the results of comparisons.
- M.5.NBT.3a.3 Identify comparison symbols. Examples: >, =, and <

M.5.NBT.4 use place value understanding to round decimals to any place. (CCSS Math.5.NBT.4)

- Round multi-digit whole numbers to any place.
- Round whole numbers to the nearest 10 or 100.
- Recognize that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones.
Perform operations with multi-digit whole numbers and with decimals to hundredths.

M.5.NBT.5 fluently multiply multi-digit whole numbers using the standard algorithm. (CCSS Math.5.NBT.5)
- Demonstrate steps in setting up a long multiplication problem.
- Multiply 2-digit numbers by 1-digit multiplier.
- Multiply 1-digit numbers by 1-digit multiplier.
- Recall basic multiplication facts.
- Recall repeated addition facts.

M.5.NBT.6 find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations and/or the relationship between multiplication and division, illustrate and explain the calculation by using equations, rectangular arrays and/or area models. (CCSS Math.5.NBT.6)
- Contrast a division equation with an example of the division algorithm.
- Illustrate the division algorithm using a one-digit divisor and a 2-digit dividend.
- Recall the properties of operations in division.
- Identify the place value of a division problem.
- Restate the inverse process of division as multiplication.
- Demonstrate the relationship between an array and multiplication.
- Recall basic multiplication facts.

M.5.NBT.7 add, subtract, multiply and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations and/or the relationship between addition and subtraction, relate the strategy to a written method and explain the reasoning used. (CCSS Math.5.NBT.7)
- Use decimal notation for fractions with denominators 10 or 100.
  Example: Rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram.
- Multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 x 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations.
- Add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.
- Apply properties of operations as strategies to multiply and divide.
- Explain why addition and subtraction strategies work, using place value and the properties of operations.
- Identify that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones.
- Identify that 100 can be thought of as a bundle of ten tens, called a “hundred.”
- Identify that the numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).
- Recall basic addition, subtraction, multiplication, and division facts.

Number & Operations - Fractions

<table>
<thead>
<tr>
<th>Performance Descriptors</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Distinguished</strong></td>
</tr>
<tr>
<td>Fifth grade students at the distinguished level in mathematics:</td>
</tr>
<tr>
<td>when given a solution, students work backwards to create a problem situation and justify their thinking; create multiple models to solve real-world problems; defend strategies and appropriateness of models chosen.</td>
</tr>
</tbody>
</table>
Use equivalent fractions as a strategy to add and subtract fractions.

M.5.NF.1 add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.) (CCSS Math.5.NF.1)

- Recall a fraction a/b with a > 1 as a sum of fractions 1/b.
- Recall addition and subtraction of fractions as joining and separating parts referring to the same whole.
- Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model.
- Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.
- Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.
- Identify two fractions as equivalent (equal) if they are the same size or the same point on a number line.
- Recognize and generate simple equivalent fractions. Explain why the fractions are equivalent, e.g., by using a visual fraction model.
- Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers.
- Compare two fractions with the same numerator or the same denominator by reasoning about their size.
- Recall basic addition, subtraction, multiplication, and division facts.

M.5.NF.2 solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem and use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2. (CCSS Math.5.NF.2)

- Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.
- Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators or by comparing to a benchmark fraction such as 1/2.
- Recognize that comparisons are valid only when the two fractions refer to the same whole.
- Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.
- Compare two fractions with the same numerator or the same denominator by reasoning about their size.
- Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.
- Recognize a fraction as a number on the number line; represent fractions on a number line diagram.
- Recognize key terms to solve word problems.
- Apply properties of operations for addition and subtraction.
- Recall basic addition and subtraction facts.

Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

M.5.NF.3 interpret a fraction as division of the numerator by the denominator (a/b = a ÷ b) and solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. For example, interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3 and that when 3 wholes are shared equally among 4 people each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie? (CCSS Math.5.NF.3)

- Define mixed number.
- Recognize a fraction a/b with a > 1 as a sum of fractions 1/b.
- Identify that fraction a/b is equivalent to fraction (na)/(nb) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size.
- Generate equivalent fractions.
- Recognize a fraction as a number on the number line; represent fractions on a number line diagram.
- Identify fraction 1/b as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts and size 1/b.

M.5.NF.4 apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction

a. interpret the product (a/b) × q as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations a × q ÷ b. For example, use a visual fraction model to show (2/3) × 4 = 8/3 and create a story context for this equation. Do the same with (2/3) × (4/5) = 8/15. (In general, (a/b) × (c/d) = ac/bd.)

b. find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths and show that the area is the same as would be found by multiplying the side lengths, multiply fractional side lengths to find areas of rectangles and represent fraction products as rectangular areas.
(CCSS Math.5.NF.4)
• Define proper fraction.
• Multiply fractions using denominators between 2 and 5.
• Identify proper and improper fractions.
• Recall basic multiplication facts.

a. Interpret the product \((a/b) \times q\) as a parts of a partition of \(q\) into \(b\) equal parts; equivalently, as the result of a sequence of operations \(a \times q \div b\).

Example: Use a visual fraction model to show \((2/3) \times 4 = 8/3\), and create a story context for this equation. Do the same with \((2/3) \times (4/5) = 8/15\). (In general, \((a/b) \times (c/d) = ac/bd\).)

▫ Model changing a whole number to a fraction.
▫ Partition a rectangle into rows and columns of same-size squares, and count to find the total number of them.
▫ Label the numerator and denominator of a fraction.

b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.

▫ Count the area squares for the length and width.
▫ Identify the width and length of a rectangle.

M.5.NF.5 interpret multiplication as scaling (resizing), by:

a. comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication,

b. explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case), explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number and relating the principle of fraction equivalence \(a/b = (n \times a)/(n \times b)\) to the effect of multiplying \(a/b\) by 1.

(CCSS Math.5.NF.5)
• Define scaling.
• Define principle of fraction equivalence.
• Multiply a fraction by a whole number.
• Compare two fractions with the same numerator or the same denominator by reasoning about their size.
• Recognize that comparisons are valid only when the two fractions refer to the same whole.
• Record the results of comparisons with the symbols \(>, =,\) or \(<\), and justify the conclusions, e.g., by using a visual fraction model.
• Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers.
• Identify factor and product.
• Use comparison symbols. Examples: \(>, =,\) or \(<\)

M.5.NF.6 solve real-world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem. (CCSS Math.5.NF.6)

• Define improper fraction, mixed number, fraction, equations, numerator, denominator.
• Multiply proper fractions with common denominators 2-10.
• Solve problems using whole numbers.
• Write number sentences for word problems.
• Identify key terms to solve multiplication word problems.
• For example: times, every, at this rate, each, per, equal/equally, in all, total
• Recall basic multiplication facts.

M.5.NF.7 apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions (Students able to multiply fractions in general can develop strategies to divide fractions in general, by reasoning about the relationship between multiplication and division. But division of a fraction by a fraction is not a requirement at this grade.)

a. interpret division of a unit fraction by a non-zero whole number and compute such quotients. For example, create a story context for \((1/3) \div 4\) and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that \((1/3) \div 4 = 1/12\) because \((1/12) \times 4 = 1/3\).

b. interpret division of a whole number by a unit fraction and compute such quotients. For example, create a story context for \(4 \div (1/5)\) and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that \(4 \div (1/5) = 20\) because \(20 \times (1/5) = 4\).

c. solve real-world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. For example, how much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many \(1/3\)-cup servings are in \(2\) cups of raisins?
(CCSS Math.5.NF.7)

- Define quotient.
- Multiply a fraction by a whole number.
- Recognize key terms to solve word problems.
- Examples: times, every, at this rate, each, per, equal/equally, in all, total
- Recall basic multiplication and division facts.

a. Interpret division of a unit fraction by a nonzero whole number, and compute such quotients.
   Example: Create a story context for \( \frac{1}{3} \div 4 \), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that \( \frac{1}{3} \div 4 = \frac{1}{12} \) because \( \frac{1}{12} \times 4 = \frac{1}{3} \).

b. Interpret division of a whole number by a unit fraction, and compute such quotients.
   Example: Create a story context for \( 4 \div \frac{1}{5} \), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that \( 4 \div \frac{1}{5} = 20 \) because \( 20 \times \frac{1}{5} = 4 \).

- Recognize a multiple of \( \frac{a}{b} \) as a multiple of \( \frac{1}{b} \), and use this understanding to multiply a fraction by a whole number.
- Recognize fraction \( \frac{a}{b} \) as a multiple of \( \frac{1}{b} \).
- Express whole numbers as fractions.
- Recognize fractions that are equivalent to whole numbers.
- Recall basic multiplication and division facts.

- Solve real-world problems involving division of unit fractions by nonzero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem.
   Examples: How much chocolate will each person get if 3 people share \( \frac{1}{2} \) lb of chocolate equally? How many \( \frac{1}{3} \)-cup servings are in 2 cups of raisins?

- Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem.

- Recognize key terms to solve word problems.
- Examples: times, every, at this rate, each, per, equal/equally, in all, total
- Recall basic multiplication and division facts.

Measurement & Data

<table>
<thead>
<tr>
<th>Performance Descriptors</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Distinguished</strong></td>
</tr>
<tr>
<td>Fifth grade students at the distinguished level in mathematics: justify solutions and create representations of real-world problems involving measurement conversions; pose new problems requiring collection and analysis of data; apply and justify multiplicative reasoning to determine possible dimensions of figures with a given volume.</td>
</tr>
</tbody>
</table>

Convert like measurement units within a given measurement system.

M.5.MD.1 convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m) and use these conversions in solving multi-step, real-world problems. (CCSS Math.5.MD.1)

- Identify relative sizes of measurement units within one system of units, including km, m, cm; kg, g; lb, oz; l, ml; and hr, min, sec.
- Express measurements in a larger unit in terms of a smaller unit.
- Solve two-step word problems.
- Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l).
- Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem.
- Recall basic addition, subtraction, multiplication, and division facts.
Represent and interpret data.
M.5.MD.2 make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8) and use operations on fractions for this grade to solve problems involving information presented in line plots. For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally. (CCSS Math.5.MD.2)
- Make a line plot to display a data set of measurements in fractions of a unit.
- Solve problems involving addition and subtraction of fractions by using information presented in line plots.
- Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories.
- Solve one- and two-step “how many more” and “how many less” problems using information presented in scaled bar graphs.
- Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories.
- Solve simple put-together, take-apart, and compare problems using information presented in a bar graph.

Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.
M.5.MD.3 recognize volume as an attribute of solid figures and understand concepts of volume measurement
a. a cube with side length 1 unit, called a “unit cube,” is said to have “one cubic unit” of volume and can be used to measure volume,
b. a solid figure which can be packed without gaps or overlaps using n unit cubes is said to have a volume of n cubic units. (CCSS Math.5.MD.3)
- Define volume including the formulas $V = l \times w \times h$, and $V = b \times h$.
- Define solid figures.
- Define unit cube.
- Recognize that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals).
- Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.
- Describe attributes of three-dimensional figures.
- Describe attributes of two-dimensional figures.
- Compare the unit size of volume/capacity in the metric system including milliliters and liters.

M.5.MD.4 measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft and improvised units. (CCSS Math.5.MD.4)
- Define cubic inches, cubic centimeters, and cubic feet.
- Compare the unit size of volume/capacity in the customary system including fluid ounces, cups, pints, quarts, gallons.
- Measure areas by counting unit squares (square cm, square in, square ft, and improvised units).
- Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l).
- Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem.
- Recall basic multiplication facts.
- Fluently add.

M.5.MD.5 relate volume to the operations of multiplication and addition and solve real-world and mathematical problems involving volume
a. find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base and represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.
- apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole number edge lengths in the context of solving real-world and mathematical problems.
- recognize volume as additive and find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real-world problems.
(CCSS Math.5.MD.5)
• Define volume.
• Apply the area and perimeter formulas for rectangles in real-world and mathematical problems.
• Solve real-world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.
• Recognize the formula for volume.
• Recall the attributes of three-dimensional solids.
• Recall basic multiplication facts.
• Fluently add.

a. Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.
   • Compare the unit size of volume/capacity in the metric system including milliliters and liters.
   • Measure and estimate liquid volumes.
   • Recall basic multiplication facts.

b. Apply the formulas \( V = l \times w \times h \) and \( V = B \times h \) for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real-world and mathematical problems.
   • Recognize the formula for volume.
   • Recall basic multiplication facts.

b. Recognize volume as additive. Find volumes of solid figures composed of two nonoverlapping right rectangular prisms by adding the volumes of the nonoverlapping parts, applying this technique to solve real-world problems.
   • Describe attributes of three-dimensional figures.
   • Describe attributes of two-dimensional figures.
   • Identify solid figures.

**Geometry**

<table>
<thead>
<tr>
<th>Performance Descriptors</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Distinguished</strong></td>
</tr>
<tr>
<td>Fifth grade students at the distinguished level in mathematics: justify solutions to problems involving points on a coordinate plane; create multiple examples and counterexamples of two-dimensional figures when given a set of attributes and justify thinking.</td>
</tr>
<tr>
<td><strong>Above Mastery</strong></td>
</tr>
<tr>
<td>Fifth grade students at the above mastery level in mathematics: create problems involving points on a coordinate plane; select examples and counterexamples of two-dimensional figures based on a given a set of attributes.</td>
</tr>
<tr>
<td><strong>Mastery</strong></td>
</tr>
<tr>
<td>Fifth grade students at the mastery level in mathematics: represent problems on a coordinate plane and find solutions; use precise language to describe, classify and identify relationships among two-dimensional figures based on attributes.</td>
</tr>
<tr>
<td><strong>Partial Mastery</strong></td>
</tr>
<tr>
<td>Fifth grade students at the partial mastery level in mathematics: given coordinates plot points in Quadrant I; describe and classify shape based on dimensional attributes.</td>
</tr>
<tr>
<td><strong>Novice</strong></td>
</tr>
<tr>
<td>Fifth grade students at the novice level in mathematics: state the coordinates of a given point; sort two-dimensional figures based on attributes.</td>
</tr>
</tbody>
</table>

**Concrete Geometric Representation (Physical Modeling)**
M.5.G.1 use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates and understand that the first number indicates how far to travel from the origin in the direction of one axis and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., x-axis and x-coordinate, y-axis and y-coordinate).
(CCSS Math.5.G.1)
• Define ordered pair of numbers.
• Define x-axis, y-axis, and zero on a coordinate.
• Specify locations on the coordinate system.
• Illustrate vertical and horizontal number lines.
• Label x- and y-axis and zero on a coordinate.
• Locate negative numbers on a horizontal number line.
• Locate positive numbers on a horizontal number line.
• Locate positive numbers on a vertical number line.
Examples: thermometer, map
M.5.G.2 represent real-world and mathematical problems by graphing points in the first quadrant of the coordinate plane and interpret coordinate values of points in the context of the situation. (CCSS Math.5.G.2)
  - Define ordered pair of numbers, quadrant one, coordinate plane, and plot points.
  - Label the horizontal axis (x).
  - Label the vertical axis (y).
  - Identify the x and y values in ordered pairs.
  - Model writing ordered pairs.

Abstract Geometric Representation (Matrix Modeling)
M.5.G.3 understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles. (CCSS Math.5.G.3)
  - Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals).
  - Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.
  - Recall attributes of shapes.
  - Sort shapes into categories.

M.5.G.4 classify two-dimensional figures in a hierarchy based on properties. (CCSS Math.5.G.4)
  - Define vertex/vertices and angle.
  - Identify that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals).
  - Recognize and draw shapes having specified attributes such as a given number of angles or a given number of equal faces.
  - Identify triangles, quadrilaterals, pentagons, hexagons, heptagons, and octagons based on the number of sides, angles, and vertices.
Content Emphases by Cluster--Grade 6*

Not all of the content in a given grade is emphasized equally in the standards. Some clusters require greater emphasis than the others based on the depth of the ideas, the time that they take to master, and/or their importance to future mathematics or the demands of college and career readiness. In addition, an intense focus on the most critical material at each grade allows depth in learning, which is carried out through the Standards for Mathematical Practice.

To say that some things have greater emphasis is not to say that anything in the standards can safely be neglected in instruction. Neglecting material will leave gaps in student skill and understanding and may leave students unprepared for the challenges of a later grade. The following table identifies the Major Clusters, Additional Clusters, and Supporting Clusters for this grade.

Key: ■ Major Clusters; □ Supporting Clusters; □ Additional Clusters

Ratios and Proportional Reasoning
■ Understand ratio concepts and use ratio reasoning to solve problems.

The Number System
■ Apply and extend previous understandings of multiplication and division to divide fractions by fractions.
□ Compute fluently with multi-digit numbers and find common factors and multiples.
■ Apply and extend previous understandings of numbers to the system of rational numbers.

Expressions and Equations
■ Apply and extend previous understandings of arithmetic to algebraic expressions.
■ Reason about and solve one-variable equations and inequalities.
■ Represent and analyze quantitative relationships between dependent and independent variables.

Geometry
■ Solve real-world and mathematical problems involving area, surface area, and volume.

Statistics and Probability
□ Develop understanding of statistical variability.
□ Summarize and describe distributions.

*Emphases are given at the cluster level. Refer to the Common Core State Standards for Mathematics for the specific standards that fall within each cluster.
In Grade 6, instructional time should focus on four critical areas: (1) connecting ratio and rate to whole number multiplication and division and using concepts of ratio and rate to solve problems; (2) completing understanding of division of fractions and extending the notion of number to the system of rational numbers, which includes negative numbers; (3) writing, interpreting and using expressions and equations; and (4) developing understanding of statistical thinking.

1. Students use reasoning about multiplication and division to solve ratio and rate problems about quantities. By viewing equivalent ratios and rates to solve problems; (2) completing understanding of division of fractions and extending the notion of number to the system of rational numbers, which includes negative numbers; (3) writing, interpreting and using expressions and equations; and (4) developing understanding of statistical thinking.

2. Students use the meaning of fractions, the meanings of multiplication and division and the relationship between multiplication and division to understand and explain why the procedures for dividing fractions make sense. Students use these operations to solve problems. Students extend their previous understandings of number and the ordering of numbers to the full system of rational numbers, which includes negative rational numbers and in particular negative integers. They reason about the order and absolute value of rational numbers and about the location of points in all four quadrants of the coordinate plane.

3. Students understand the use of variables in mathematical expressions. They write expressions and equations that correspond to given situations, evaluate expressions and use expressions and formulas to solve problems. Students understand that expressions in different forms can be equivalent, and they use the properties of operations to rewrite expressions in equivalent forms. Students know that the solutions of an equation are the values of the variables that make the equation true. Students use properties of operations and the idea of maintaining the equality of both sides of an equation to solve simple one-step equations. Students construct and analyze tables, such as tables of quantities that are in equivalent ratios, and they use equations (such as 3x = y) to describe relationships between quantities.

4. Building on and reinforcing their understanding of number, students begin to develop their ability to think statistically. Students recognize that a data distribution may not have a definite center and that different ways to measure center yield different values. The median measures center in the sense that it is the middle value. The mean measures center in the sense that it is the value that each data point would take on if the total of the data values were redistributed equally, and also in the sense that it is a balance point. Students recognize that a measure of variability (interquartile range or mean absolute deviation) can also be useful for summarizing data because two very different sets of data can have the same mean and median yet be distinguished by their variability. Students learn to describe and summarize numerical data sets, identifying clusters, peaks, gaps and symmetry, considering the context in which the data were collected.

Students in Grade 6 also build on their work with area in elementary school by reasoning about relationships among shapes to determine area, surface area, and volume. They find areas of right triangles, other triangles, and special quadrilaterals by decomposing these shapes, rearranging or removing pieces and relating the shapes to rectangles. Using these methods, students discuss, develop and justify formulas for areas of triangles and parallelograms. Students find areas of polygons and surface areas of prisms and pyramids by decomposing them into pieces whose area they can determine. They reason about right rectangular prisms with fractional side lengths to extend formulas for the volume of a right rectangular prism to fractional side lengths. They prepare for work on scale drawings and constructions in Grade 7 by drawing polygons in the coordinate plane.

**Mathematical Practices**

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

<table>
<thead>
<tr>
<th>Grade</th>
<th>Focus Areas in Support of Rich Instruction and Expectations of Fluency and Conceptual Understanding</th>
</tr>
</thead>
<tbody>
<tr>
<td>K-2</td>
<td>Addition and subtraction, measurement using whole number quantities</td>
</tr>
<tr>
<td>3-5</td>
<td>Multiplication and division of whole numbers and fractions</td>
</tr>
<tr>
<td>6</td>
<td>Ratios and proportional reasoning; early expressions and equations</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Grade</th>
<th>Standard</th>
<th>Required Fluencies</th>
</tr>
</thead>
<tbody>
<tr>
<td>K</td>
<td>K.OA.5</td>
<td>Add/subtract within 5</td>
</tr>
<tr>
<td>1</td>
<td>1.OA.6</td>
<td>Add/subtract within 10</td>
</tr>
<tr>
<td>2</td>
<td>2.OA.2</td>
<td>Add/subtract within 20 (know single-digit sums from memory)</td>
</tr>
<tr>
<td></td>
<td>2.NBT.5</td>
<td>Add/subtract within 100</td>
</tr>
<tr>
<td>3</td>
<td>3.OA.7</td>
<td>Add/subtract within 100 (know single-digit products from memory)</td>
</tr>
<tr>
<td></td>
<td>3.NBT.2</td>
<td>Add/subtract within 1,000</td>
</tr>
<tr>
<td>4</td>
<td>4.NBT.4</td>
<td>Add/subtract within 1,000,000</td>
</tr>
<tr>
<td>5</td>
<td>5.NBT.5</td>
<td>Multi-digit multiplication</td>
</tr>
<tr>
<td>6</td>
<td>6.NS.2.3</td>
<td>Multi-digit division</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Multi-digit decimal operations</td>
</tr>
</tbody>
</table>
Ratios & Proportional Relationships

<table>
<thead>
<tr>
<th>Performance Descriptors</th>
<th>Distinguished</th>
<th>Above Mastery</th>
<th>Mastery</th>
<th>Partial Mastery</th>
<th>Novice</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sixth grade students at the distinguished level in mathematics: analyze ratios and the relationship to fractions and communicate similarities and differences.</td>
<td>Sixth grade students at the above mastery level in mathematics: create and model problems requiring ratio or proportional reasoning.</td>
<td>Sixth grade students at the mastery level in mathematics: state the meaning of ratio concepts, use ratio reasoning and rates to solve problems.</td>
<td>Sixth grade students at the partial mastery level in mathematics: write ratios describing a relationship between two quantities and make tables of equivalent ratios.</td>
<td>Sixth grade students at the novice level in mathematics: recognize and utilize ratios.</td>
<td></td>
</tr>
</tbody>
</table>

Understand ratio concepts and use ratio reasoning to solve problems.

M.6.RP.1 understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. For example, “The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak.” “For every vote candidate A received, candidate C received nearly three votes.” (CCSS Math.6.RP.1)

- Define quantity, fraction, and ratio.
- Reinterpret a fraction as a ratio. Example: Read 2/3 as 2 out of 3.
- Write a ratio as a fraction.
- Draw a model of a given fraction.
- Identify the numerator and denominator of a fraction.

M.6.RP.2 understand the concept of a unit rate a/b associated with a ratio a:b with b ≠ 0, and use rate language in the context of a ratio relationship. For example, “This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar.” “We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger.” (Expectations for unit rates in this grade are limited to non-complex fractions.) (CCSS Math.6.RP.2)

- Define unit rate, proportion, and rate.
- Create a ratio or proportion from a given word problem.
- Calculate unit rate by using ratios or proportions.
- Interpret a fraction as division of the numerator by the denominator.
- Example: (a/b=a÷b).
- Write a ratio as a fraction.

M.6.RP.3 use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

- Make tables of equivalent ratios relating quantities with whole number measurements, find missing values in the tables and plot the pairs of values on the coordinate plane. Use tables to compare ratios.
- Solve unit rate problems including those involving unit pricing and constant speed. For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?
- Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.
- Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

(CCSS Math.6.RP.3)

- Create a ratio or proportion from a given word problem, diagram, table, or equation.
- Calculate unit rate or rate by using ratios or proportions.
- Restate real world problems or mathematical problems.

a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.
- Define coordinate plane, equivalent, input, output, and ordered pairs.
- Construct a graph from a set of ordered pairs given in the table of equivalent ratios.
- Calculate missing input and/or output values in a table.
- Draw and label a table of equivalent ratios from given information.
- Identify the parts of a table of equivalent ratios (input, output, etc).

b. Solve unit rate problems including those involving unit pricing and constant speed. Example: If it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?
- Compute the unit rate, unit price, and constant speed.
- Create a proportion or ratio from a given word problem.
- Identify the two units being compared.
c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity measure of centers 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.
   ▪ Define percent.
   ▪ Calculate a proportion for missing information.
   ▪ Identify a proportion from given information.
   ▪ Solve a proportion using part over whole equals percent over 100.
d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.
   ▪ Form a ratio.
   ▪ Convert like measurement units within a given system.
   ▪ Example: $120 \text{ min} = 2 \text{ hrs}$
   ▪ Know relative sizes of measurement units within one system of units, including km, m, cm; kg, g; lb, oz; l, ml; and hr, min, sec.

### The Number System

<table>
<thead>
<tr>
<th>Performance Descriptors</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Distinguished</strong></td>
</tr>
<tr>
<td>Sixth grade students at the distinguished level in mathematics:</td>
</tr>
<tr>
<td>communicate understanding of the connections among problems, models and numerical solutions created;</td>
</tr>
<tr>
<td>communicate understanding of the connection between the greatest common factor and the distributive property; reason inductively that linear relationships exist and communicate how graphs can be used to model problems.</td>
</tr>
</tbody>
</table>

Apply and extend previous understandings of multiplication and division to divide fractions by fractions.

M.6.NS.1 Interpret and compute quotients of fractions and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. For example, create a story context for $\frac{2}{3} \div \frac{3}{4}$ and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that $\left(\frac{2}{3}\right) \div \left(\frac{3}{4}\right) = \frac{8}{9}$ because $\frac{3}{4}$ of $\frac{8}{9}$ is $\frac{2}{3}$. (In general, $\left(\frac{a}{b}\right) \div \left(\frac{c}{d}\right) = \frac{ad}{bc}$.)

How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt?

How wide is a rectangular strip of land with length 3/4 mi and area ½ square mi? (CCSS Math.6.NS.1)

- Define fraction (including numerator and denominator), reciprocal, equation, and quotient.
- Construct an equation from a given word problem.
- Interpret division of fractions by changing division to multiplication.
- Demonstrate division of fractions using a visual fraction model.
- Discuss the process for multiplying fractions.

Compute fluently with multi-digit numbers and find common factors and multiples.

M.6.NS.2 Fluently divide multi-digit numbers using the standard algorithm. (CCSS Math.6.NS.2)

- Define factors and multiples.
- Discuss the steps for solving a division problem.
- Recognize division and multiplication as inverse operations.
- Recall basic division and multiplication facts.
M.6.NS.3 fluently add, subtract, multiply and divide multi-digit decimals using the standard algorithm for each operation. (CCSS Math.6.NS.3)
- Solve division problems involving multi-digit whole numbers and decimal numbers.
- Solve multiplication problems involving multi-digit whole numbers and decimal numbers.
- Recall basic multiplication and division facts.
- Solve addition and subtraction of multi-digit decimal numbers (emphasis on alignment)
- Solve addition and subtraction of multi-digit whole numbers.
- Recognize place value of whole numbers and decimals.
- Demonstrate addition, subtraction, multiplication, and division of whole numbers and decimals using manipulatives.

M.6.NS.4 find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. For example, express 36 + 8 as 4 (9 + 2). (CCSS Math.6.NS.4)
- Define greatest common factor, least common multiple, and the distributive property.
- Design problems using greatest common factor and the distributive property.
- Show an understanding of how to solve a problem using the distributive property.
- Identify the least common multiple of a given set of numbers.
- List multiples of any given whole number.
- Identify the greatest common factors of a given set of numbers.
- List common factors of given whole numbers.

Apply and extend previous understandings of numbers to the system of rational numbers.
M.6.NS.5 understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation. (CCSS Math.6.NS.5)
- Define integers, positive and negative numbers.
- Demonstrate the location of positive and negative numbers on a vertical and horizontal number line.
- Give examples of positive and negative numbers to represent quantities having opposite directions in real-world contexts.
- Discuss the measure of centering of 0 in relationship to positive and negative numbers.

M.6.NS.6 understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.

a. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., \(-(-3) = 3\), and that 0 is its own opposite.
- Discover that the opposite of the opposite of a number is the number itself.
- Show on a number line that numbers that are equal distance from 0 and on opposite sides of 0 have opposite signs.

b. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.
- Demonstrate when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.

M.6.NS.6 understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.

a. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., \(-(-3) = 3\), and that 0 is its own opposite.
- Discover that the opposite of the opposite of a number is the number itself.
- Show on a number line that numbers that are equal distance from 0 and on opposite sides of 0 have opposite signs.

b. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.
- Demonstrate when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.

M.6.NS.6 understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.

a. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., \(-(-3) = 3\), and that 0 is its own opposite.
- Discover that the opposite of the opposite of a number is the number itself.
- Show on a number line that numbers that are equal distance from 0 and on opposite sides of 0 have opposite signs.

b. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.
- Demonstrate when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.
M.6.NS.7 understand ordering and absolute value of rational number.

a. Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. For example, interpret –3 > –7 as a statement that –3 is located to the right of –7 on a number line oriented from left to right.

b. Write, interpret, and explain statements of order for rational number in real-world contexts. For example, write –3°C > –7°C to express the fact that –3°C is warmer than –7°C.

c. Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. For example, for an account balance of –30 dollars, write |–30| = 30 to describe the size of the debt in dollars.

d. Distinguish comparisons of absolute value from statements about order. For example, recognize that an account balance less than –30 dollars represents a debt greater than 30 dollars.

(CCSS Math.6.NS.7)

• Define absolute value.
• Recall how to order numbers.

a. Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram.

Example: Interpret –3 > –7 as a statement that –3 is located to the right of –7 on a number line oriented from left to right.

• Define inequality.
• Explain the inequality of two numbers using their position on a number line in relation to 0.

b. Write, interpret, and explain statements of order for rational numbers in real-world contexts.

Example: Write –3°C > –7°C to express the fact that –3°C is warmer than –7°C.

• Explain how to apply statements of order for rational numbers in real-world contexts.
• Restate the measure of centering of a statement of order for rational numbers in real-world context.
• Write a statement of order for rational numbers in real-world contexts.
• Recall the measure of centering of greater than and less than and their symbols.

c. Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation.

Example: For an account balance of –30 dollars, write |–30| = 30 to describe the size of the debt in dollars.

• Give examples of the magnitude for a positive or negative quantity in a real-world situations using absolute value.
• Recognize the absolute value of a rational number is its’ distance from 0 on a vertical and horizontal number line.

d. Distinguish comparisons of absolute value from statements about order.

Example: Recognize that an account balance less than –30 dollars represents a debt greater than 30 dollars.

• Evaluate a statement about order using comparisons of absolute value.
• Recall how to order positive and negative numbers. (Use number line if needed)

M.6.NS.8 solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate. (CCSS Math.6.NS.8)

• Calculate the distances between points having the same first or second coordinate using absolute value.
• Interpret graphing points in all four quadrants of the coordinate plane in real-world situations.
• Recall how to graph points in all four quadrants of the coordinate plane.

### Expressions & Equations

<table>
<thead>
<tr>
<th>Performance Descriptors</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Distinguished</strong></td>
</tr>
<tr>
<td>Sixth grade students at the distinguished level in mathematics: create real-life representations of given algebraic expressions; identify and communicate constraints on variables based on the context of the equation or inequality; identify a real-world problem which models using dependent and independent variables, collect and analyze data and find solutions.</td>
</tr>
<tr>
<td><strong>Above Mastery</strong></td>
</tr>
<tr>
<td>Sixth grade students at the above mastery level in mathematics: translate algebraic expressions into words and state the meanings of symbols; create and identify pathways to solve real-world problems requiring algebraic expressions or inequalities; create and identify pathways to solve real-world problems involving dependent and independent variables.</td>
</tr>
<tr>
<td><strong>Mastery</strong></td>
</tr>
<tr>
<td>Sixth grade students at the mastery level in mathematics: extend reasoning from numerical to algebraic expressions, identify and simplify equivalent expressions and communicate meaning using appropriate mathematical vocabulary; use algebraic equations and inequalities to solve real-world problems and understand domains and meanings of variables in different contexts; analyze relationships between dependent and independent variables, state the meaning of variables, write applicable equations, and analyze using graphs and tables.</td>
</tr>
<tr>
<td><strong>Partial Mastery</strong></td>
</tr>
<tr>
<td>Sixth grade students at the partial mastery level in mathematics: identify terms in a given algebraic expression; evaluate algebraic expressions and inequalities using substitution; identify dependent and independent variables in context.</td>
</tr>
<tr>
<td><strong>Novice</strong></td>
</tr>
<tr>
<td>Sixth grade students at the novice level in mathematics: evaluate algebraic expressions when given a value for the variable; evaluate algebraic equations using number lines; recognize when a word problem requires two variables.</td>
</tr>
</tbody>
</table>
Apply and extend previous understandings of arithmetic to algebraic expressions.

M.6.EE.1 write and evaluate numerical expressions involving whole-number exponents. (CCSS Math.6.EE.1)
   • Define exponent, numerical expression, algebraic expression, variable, base, power, square of a number, and cube of a number.
   • Compute a numerical expression with exponents.
   • Restate exponential numbers as repeated multiplication.
   • Choose the correct value to replace each variable in the expression (Substitution).
   • Calculate the multiplication of single or multi digit whole numbers.

M.6.EE.2 write, read and evaluate expressions in which letters stand for numbers.
   a. Write expressions that record operations with numbers and with letters standing for numbers. For example, express the calculation “Subtract y from 5” as 5 – y.
   b. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. For example, describe the expression 2(8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms.
   c. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). For example, use the formulas V = s³ and A = 6s² to find the volume and surface area of a cube with sides of length s = 1/2.
   (CCSS Math.6.EE.2)
   • Define algebraic expression and variable.
   • Convert mathematical term to mathematical symbols and numbers.
   Example: Express the calculation, “Subtract y from 5,” as 5 – y.
   ▫ Convert mathematical terms to mathematical symbols and numbers (Ex. sum; +, difference; -, product; ●, quotient; ÷).
   ▫ Recall different ways to show multiplication and division.
   b. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity.
   Example: Describe the expression 2(8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms.
   ▫ Define coefficient and term.
   ▫ Match mathematical terms with correct mathematical symbols.
   c. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations).
   ▫ Calculate an expression in the correct order (Ex. exponents, mult./div. from left to right, and add/sub. from left to right).
   ▫ Choose the correct value to replace each variable in the algebraic expression (Substitution).
   ▫ Calculate a numerical expression (Ex. V=4●4●4).
   ▫ Recognize the correct order to solve expressions with more than one operation.

M.6.EE.3 apply the properties of operations to generate equivalent expressions. For example, apply the distributive property to the expression 3 (2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6 (4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y. (CCSS Math.6.EE.3)
   • Define equivalent, simplify, term, distributive property, associative property of addition and multiplication, and the commutative property of addition and multiplication.
   • Simplify expressions with parenthesis (Ex. 5(4 + x) = 20 + 5x).
   • Combine terms that are alike of a given expression.
   • Recognize the property demonstrated in a given expression.
   • Simplify an expression by dividing by the greatest common factor.
   Example: 18x + 6y = 6(3x + y).
   • Determine the greatest common factor.

M.6.EE.4 identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for. (CCSS Math.6.EE.4)
   • Define equivalent expressions.
   • Recognize equivalent expressions.
   • Substitute for the variable to find the value of a given expression.
   • Calculate a numerical expression (Ex. V=4●4●4).
   • Recognize that a variable without a written coefficient is understood to be one.
Reason about and solve one-variable equations and inequalities.

M.6.EE.5 understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true. (CCSS Math.6.EE.5)

- Define equation, solution of an equation, solution of an inequality, and inequality.
- Compare and contrast equations and inequalities.
- Determine if an inequality is by replacing the variable with a given number.
- Determine if an equation is true by replacing the variable with a given number.
- Simplify a numerical sentence to determine equivalence.
- Recognize the symbols for \(=, >, <, \leq, \text{ and } \geq\).

M.6.EE.6 use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number or depending on the purpose at hand, any number in a specified set. (CCSS Math.6.EE.6)

- Define equation and variable.
- Set up an equation to represent the given situation, using correct mathematical operations and variables.
- Identify the unknown, in a given situation, as the variable.
- List given information from the problem.

M.6.EE.7 solve real-world and mathematical problems by writing and solving equations of the form \(x + p = q\) and \(px = q\) for cases in which \(p, q\) and \(x\) are all nonnegative rational numbers. (CCSS Math.6.EE.7)

- Define equation and variable.
- Solve the equation represented by the real-world situation.
- Set up an equation to represent the given situation, using correct mathematical operations and variables.
- Identify the unknown variable in a given situation.
- List given information from the problem.

M.6.EE.8 write an inequality of the form \(x > c\) or \(x < c\) to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form \(x > c\) or \(x < c\) have infinitely many solutions; represent solutions of such inequalities on number line diagrams. (CCSS Math.6.EE.8)

- Define inequality and solution set of an inequality.
- Set up an inequality to represent the given situation, using correct mathematical operations and variables.
- Identify solution set for the inequality used to represent the situation.
- Graph the solution set on a number line for the inequality used to represent the situation.
- Recognize the inequality symbols; \(<, >\).
- Construct and label a number line.

Represent and analyze quantitative relationships between dependent and independent variables.

M.6.EE.9 use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of another quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation \(d = 65t\) to represent the relationship between distance and time. (CCSS Math.6.EE.9)

- Define dependent variable, independent variable, ordered pairs, input, output, and coordinate plane.
- Examine the graph and table to determine any relationship between the variables.
- Analyze the pattern represented by the values in the table and develop an equation to express the relationship.
- Plot independent (input) and dependent (output) values on a coordinate plane.
- Create a table of independent and dependent values from the equation.
- Draw and label a coordinate plane.
- Recall how to draw a number line.

Geometry

<table>
<thead>
<tr>
<th>Performance Descriptors</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Distinguished</strong></td>
</tr>
<tr>
<td>Sixth grade students at the distinguished level in mathematics: construct and compare possible models of a three-dimensional figure with a given volume.</td>
</tr>
<tr>
<td><strong>Above Mastery</strong></td>
</tr>
<tr>
<td>Sixth grade students at the above mastery level in mathematics: make and test a conjecture and communicate how changing one dimension affects volume.</td>
</tr>
<tr>
<td><strong>Mastery</strong></td>
</tr>
<tr>
<td>Sixth grade students at the mastery level in mathematics: create representations of three-dimensional geometric figures while solving real-world and mathematical problems involving surface area and volume.</td>
</tr>
<tr>
<td><strong>Partial Mastery</strong></td>
</tr>
<tr>
<td>Sixth grade students at the partial mastery level in mathematics: using three-dimensional figures and given formulas, determine volume of the figures and area of any face.</td>
</tr>
<tr>
<td><strong>Novice</strong></td>
</tr>
<tr>
<td>Sixth grade students at the novice level in mathematics: identify faces of three-dimensional figures as two-dimensional geometric shapes.</td>
</tr>
</tbody>
</table>
Solve real-world and mathematical problems involving area, surface area, and volume.
M.6.G.1 find the area of right triangles, other triangles, special quadrilaterals and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems. (CCSS Math.6.G.1)
  • Define area, special quadrilaterals, right triangles, and polygons.
  • Analyze the area of other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes.
  • Apply area formulas to solve real-world mathematical problems.
  • Demonstrate how the area of a rectangle is equal to the sum of the area of two equal right triangles.
  • Explain how to find the area for rectangles.
  • Select manipulatives to demonstrate how to compose and decompose triangles and other shapes.
  • Recognize and demonstrate that two right triangles make a rectangle.
M.6.G.2 find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas V = l w h and V = B h to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems. (CCSS Math.6.G.2)
  • Define volume, rectangular prism, edge, and formula.
  • Evaluate the volumes of rectangular prisms in the context of solving real-world and mathematical problems.
  • Set up V=lwh and V=Bh to find volumes in the context of solving real-world and mathematical problems.
  • Calculate the volume of a rectangular prism using fractional lengths.
  • Test the formula V= lwh and V=Bh with the experimental findings.
  • Experiment with finding the volume using a variety of sizes of rectangular prisms.
  • Discover the volume of a rectangular prism using manipulatives.
  • Recall how to multiply fractional numbers.
M.6.G.3 draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems. (CCSS Math.6.G.3)
  • Define vertices.
  • Apply absolute value to find the length of a side joining points with the same first coordinate or the same second coordinate.
  • Plot points on a Cartesian plane, then connect points for the vertices to sketch a polygon.
  • Identify ordered pairs.
  • Recognize polygons.
M.6.G.4 represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems. (CCSS Math.6.G.4)
  • Define three-dimensional figures, surface area, and nets.
  • Evaluate how to apply using surface area of a three-dimensional figure to solving real-world and mathematical problems.
  • Draw nets to find the surface area of a given three-dimensional figure.
  • Recall how to calculate the area of a rectangle and triangle.
  • Select and create a three-dimensional figure using manipulatives.
  • Identify three-dimensional figures.

Statistics & Probability

<table>
<thead>
<tr>
<th>Performance Descriptors</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Distinguished</strong></td>
</tr>
<tr>
<td>Sixth grade students at the distinguished level in mathematics: construct a viable argument as to the best average (mean, median or mode) of a set of data and justify reasoning; construct a viable argument that describes an overall pattern of the data distribution taking into account the context from which the data arose.</td>
</tr>
</tbody>
</table>
Develop understanding of statistical variability.

M.6.SP.1 recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. For example, “How old am I?” is not a statistical question, but “How old are the students in my school?” is a statistical question because one anticipates variability in students’ ages. (CCSS Math.6.SP.1)

• Define statistical question.
• Compare and contrast statistical questions and non-statistical questions.

M.6.SP.2 understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape. (CCSS Math.6.SP.2)

• Define distribution and skew.
• Describe the shape of a set of data in a given distribution.
• Describe the spread of a set of data in a given distribution.
• Describe the center of a set of data in a given distribution.
• Recall how to read a graph or table.

M.6.SP.3 recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number. (CCSS Math.6.SP.3)

• Define numerical data set, measure of variation, and measure of center.
• Relate the measure of variation, of a data set, with the concept of range.
• Relate the measure of the center for a numerical data set with the concept of measure of center.

Summarize and describe distributions.

M.6.SP.4 display numerical data in plots on a number line, including dot plots, histograms and box plots. (CCSS Math.6.SP.4)

• Define dot plots, upper quartile, lower quartile, median, histograms, and box plots.
• Calculate upper quartile median, lower quartile median, overall median, greatest value, and lowest value.
• Create box plot using calculations.
• Plot data on dot plots and histograms.
• Construct and label the display.
• Recognize the different types of displays.

M.6.SP.5 Summarize numerical data sets in relation to their context, such as by:

a. reporting the number of observations.

b. describing the nature of the attribute under investigation, including how it was measured and its units of measurement.

c. giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.

d. relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered. (CCSS Math.6.SP.5)

• Define numerical data set, quantitative, measure of center, median, frequency distribution, and attribute.
• Explain or give a narrative description of the graph including any overall pattern and any striking deviations from the pattern.
• Compare and contrast the center and variation.
• Graph and label the set of data.
• Organize the data.
• Collect the data.
• Describe how attribute was measured including units of measurement.
• Identify the attribute used to create the numerical set.
Mathematics (M) Grade 7

Content Emphases by Cluster--Grade 7*

Not all of the content in a given grade is emphasized equally in the standards. Some clusters require greater emphasis than the others based on the depth of the ideas, the time that they take to master, and/or their importance to future mathematics or the demands of college and career readiness. In addition, an intense focus on the most critical material at each grade allows depth in learning, which is carried out through the Standards for Mathematical Practice.

To say that some things have greater emphasis is not to say that anything in the standards can safely be neglected in instruction. Neglecting material will leave gaps in student skill and understanding and may leave students unprepared for the challenges of a later grade. The following table identifies the Major Clusters, Additional Clusters, and Supporting Clusters for this grade.

Key: ■ Major Clusters; □ Supporting Clusters; □ Additional Clusters

Ratios and Proportional Reasoning
■ Analyze proportional relationships and use them to solve real-world and mathematical problems.

The Number System
■ Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

Expressions and Equations
■ Use properties of operations to generate equivalent expressions.
■ Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

Geometry
□ Draw, construct and describe geometrical figures and describe the relationships between them.
□ Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.

Statistics and Probability
■ Use random sampling to draw inferences about a population.
□ Draw informal comparative inferences about two populations.
■ Investigate chance processes and develop, use, and evaluate probability models.

*Emphases are given at the cluster level. Refer to the Common Core State Standards for Mathematics for the specific standards that fall within each cluster.
In Grade 7, instructional time should focus on four critical areas: (1) developing understanding of and applying proportional relationships; (2) developing understanding of operations with rational numbers and working with expressions and linear equations; (3) solving problems involving scale drawings and informal geometric constructions and working with two- and three-dimensional shapes to solve problems involving area, surface area and volume; and (4) drawing inferences about populations based on samples.

1. Students extend their understanding of ratios and develop understanding of proportionality to solve single- and multi-step problems. Students use their understanding of ratios and proportionality to solve a wide variety of percent problems, including those involving discounts, interest, taxes, tips and percent increase or decrease. Students solve problems about scale drawings by relating corresponding lengths between the objects or by using the fact that relationships of lengths within an object are preserved in similar objects. Students graph proportional relationships and understand the unit rate informally as a measure of the steepness of the related line, called the slope. They distinguish proportional relationships from other relationships.

2. Students develop a unified understanding of number, recognizing fractions, decimals (that have a finite or a repeating decimal representation), and percents as different representations of rational numbers. Students extend addition, subtraction, multiplication and division to all rational numbers, maintaining the properties of operations and the relationships between addition and subtraction and multiplication and division. By applying these properties and by viewing negative numbers in terms of everyday contexts (e.g., amounts owed or temperatures below zero), students explain and interpret the rules for adding, subtracting, multiplying and dividing with negative numbers. They use the arithmetic of rational numbers as they formulate expressions and equations in one variable and use these equations to solve problems.

3. Students continue their work with area from Grade 6, solving problems involving the area and circumference of a circle and surface area of three-dimensional objects. In preparation for work on congruence and similarity in Grade 8 they reason about relationships among two-dimensional figures using scale drawings and informal geometric constructions, and they gain familiarity with the relationships between angles formed by intersecting lines. Students work with three-dimensional figures, relating them to two-dimensional figures by examining cross-sections. They solve real-world and mathematical problems involving area, surface area and volume of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes and right prisms.

4. Students build on their previous work with single data distributions to compare two data distributions and address questions about differences between populations. They begin informal work with random sampling to generate data sets and learn about the importance of representative samples for drawing inferences.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.
## Ratios & Proportional Relationships

<table>
<thead>
<tr>
<th>Performance Descriptors</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Distinguished</strong></td>
</tr>
<tr>
<td>Seventh grade students at the distinguished level in mathematics: make assumptions in order to simplify a complicated situation and justify those assumptions using proportional reasoning.</td>
</tr>
</tbody>
</table>

### Analyze proportional relationships and use them to solve real-world and mathematical problems.

M.7.RP.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction 1/2/1/4 miles per hour, equivalently 2 miles per hour. (CCSS Math.7.RP.1)
- Define unit rate, proportions, area, length, and ratio.
- Recall how to find unit rates using ratios.
- Recall the steps used to solve division of fraction problems.
- Discuss the measure of centering of ratios.

M.7.RP.2 Recognize and represent proportional relationships between quantities.
- a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.
- b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.
- c. Represent proportional relationships by equations. For example, if total cost \( t \) is proportional to the number \( n \) of items purchased at a constant price \( p \), the relationship between the total cost and the number of items can be expressed as \( t = pn \).
- d. Explain what a point \((x, y)\) on the graph of a proportional relationship means in terms of the situation, with special attention to the points \((0, 0)\) and \((1, r)\) where \( r \) is the unit rate. (CCSS Math.7.RP.2)
- Define proportions and proportional relationships.
- Demonstrate how to write ratios as a fraction.
- Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.
- Define equivalent ratios and origin.
- Locate the origin on a coordinate plane.
- Show how to graph on Cartesian plane.
- Determine if the graph is a straight line through the origin.
- Use a table or graph to determine whether two quantities are proportional.
- Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.
- Define a constant and equations.
- Create a table from a verbal description, diagram, or a graph.
- Identify numeric patterns and finding the rule for that pattern.
- Recall how to find unit rate.
- Represent proportional relationships by equations.
- Example: If total cost \( t \) is proportional to the number \( n \) of items purchased at a constant price \( p \), the relationship between the total cost and the number of items can be expressed as \( t = pn \).
- Recall how to write equations to represent a proportional relationship.
- Discuss the use of variables.
- Explain what a point \((x, y)\) on the graph of a proportional relationship measure of centers in terms of the situation, with special attention to the points \((0, 0)\) and \((1, r)\) where \( r \) is the unit rate.
- Define ordered pairs.
- Show how to plot points on a Cartesian plane.
- Locate the origin on the coordinate plane.
- Discuss the measure of centering of ratios and unit rates.
M.7.RP.3 use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error. (CCSS Math.7.RP.3)

- Define interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, and percent error.
- Apply definitions to context in real world problems.
- Discuss definitions using real world examples.
- Solve proportional problems.
- Recall how to find percent and ratios.
- Recall steps for solving multi-step problems.

The Number System

<table>
<thead>
<tr>
<th>Performance Descriptors</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Distinguished</strong></td>
</tr>
<tr>
<td>Seventh grade students at the distinguished level in mathematics: create and present scenarios involving real-world situations to model understanding of the properties of operations and rational numbers.</td>
</tr>
<tr>
<td><strong>Above Mastery</strong></td>
</tr>
<tr>
<td>Seventh grade students at the above mastery level in mathematics: explain and justify the selection of strategies used to solve problems involving properties of operations and rational numbers.</td>
</tr>
<tr>
<td><strong>Mastery</strong></td>
</tr>
<tr>
<td>Seventh grade students at the mastery level in mathematics: apply properties of operations to complete computations and solve real-world problems involving rational numbers.</td>
</tr>
<tr>
<td><strong>Partial Mastery</strong></td>
</tr>
<tr>
<td>Seventh grade students at the partial mastery level in mathematics: apply the properties of operations in computations involving integers.</td>
</tr>
<tr>
<td><strong>Novice</strong></td>
</tr>
<tr>
<td>Seventh grade students at the novice level in mathematics: recognize rational numbers and their additive inverses.</td>
</tr>
</tbody>
</table>

Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

M.7.NS.1 apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

- a. Describe situations in which opposite quantities combine to make 0. For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged.
- b. Understand p + q as the number located a distance |q| from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.
- c. Understand subtraction of rational numbers as adding the additive inverse, p – q = p + (–q). Show that the distance between two rational numbers on the number line is the absolute value of their difference and apply this principle in real-world contexts.
- d. Apply properties of operations as strategies to add and subtract rational numbers. (CCSS Math.7.NS.1)

- Define rational numbers, horizontal, and vertical.
- Recall how to extend a horizontal number line.
- Recall how to extend a vertical number line.
- Demonstrate addition and subtraction of whole numbers using a horizontal or vertical number line.
- Give examples of rational numbers.
- Describe situations in which opposite quantities combine to make 0.
  - Example: A hydrogen atom has 0 charge because its two constituents are oppositely charged.
- Explain the measure of centering of 0 in representing positive and negative quantities.
- Locate positive and negative numbers on a number line.
- Recall properties of addition and subtraction.
- Understand p + q as the number located a distance |q| from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its’ opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.
- Define absolute value and additive inverse.
  - Model addition and subtraction using manipulatives.
  - Show addition and subtraction using a number line.
- Understand subtraction of rational numbers as adding the additive inverse, p – q = p + (–q). Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.
- Define absolute value and additive inverse.
- Show subtraction as the additive inverse.
- Give examples of the opposite of a given number.
- Show addition and subtraction using a number line.
- a. Apply properties of operations as strategies to add and subtract rational numbers.
  - Discuss various strategies for solving real-world and mathematical problems.
  - Identify properties of operations for addition and subtraction.
  - Recall the steps for solving addition and subtraction of rational numbers.
M.7.NS.2 apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.

a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (−1)(−1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.

b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with nonzero divisor) is a rational number. If p and q are integers, then −(p/q) = (−p)/q = p/(−q). Interpret quotients of rational numbers by describing real-world contexts.

c. Apply properties of operations as strategies to multiply and divide rational numbers.

d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.

(CCSS Math.7.NS.2)

• Recall the steps for solving multiplication and division of fraction problems.
• Recall the steps for solving multiplication and division of whole number problems.

Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (−1)(−1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.

• Define distributive property, rational numbers, product.
• Solve problems using the distributive property.
• Recall basic multiplication facts using manipulatives.
• Identify the properties of operations for multiplication.

Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with nonzero divisor) is a rational number.

• Define quotient, divisor, and integer.
• Recall the rules for multiplying integers.
• Solve real-world problems.
• Recall the steps of division.

Apply properties of operations as strategies to multiply and divide rational numbers.

• Discuss various strategies for solving real-world and mathematical problems.
• Identify properties of operations for multiplication.

Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.

• Define terminating decimals.
• Give examples of equivalent fractions and decimals.
• Recall the steps for dividing decimals.
• Recall the steps of division.

M.7.NS.3 solve real-world and mathematical problems involving the four operations with rational numbers. (Computations with rational numbers extend the rules for manipulating fractions to complex fractions.) (CCSS Math.7.NS.3)

• Discuss various strategies for solving real-world and mathematical problems.
• Recall steps for solving fractional problems.
• Identify properties of operations for addition and multiplication.
• Recall the rules for multiplication and division of rational numbers.
• Recall the rules for addition and subtraction of rational numbers.
Expressions & Equations

<table>
<thead>
<tr>
<th>Performance Descriptors</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Distinguished</strong></td>
</tr>
<tr>
<td>Seventh grade students at the distinguished level in mathematics:</td>
</tr>
<tr>
<td>create and present scenarios modeled by multiple equivalent expressions;</td>
</tr>
<tr>
<td>recognize problems in real-world situations that can be modeled and solved through the application of equations and inequalities.</td>
</tr>
<tr>
<td><strong>Above Mastery</strong></td>
</tr>
<tr>
<td>Seventh grade students at the above mastery level in mathematics:</td>
</tr>
<tr>
<td>communicate how various properties of operations justify procedures used to simplify expressions;</td>
</tr>
<tr>
<td>assess reasonableness of and justify solutions to problems involving rational numbers.</td>
</tr>
<tr>
<td><strong>Mastery</strong></td>
</tr>
<tr>
<td>Seventh grade students at the mastery level in mathematics:</td>
</tr>
<tr>
<td>use properties of operations to make sense of and modify linear expressions in the context of a problem;</td>
</tr>
<tr>
<td>generate equations and inequalities using variables to find and display solutions to multi-step problems involving rational numbers.</td>
</tr>
<tr>
<td><strong>Partial Mastery</strong></td>
</tr>
<tr>
<td>Seventh grade students at the partial mastery level in mathematics:</td>
</tr>
<tr>
<td>generate equivalent expressions through the application of properties of operations;</td>
</tr>
<tr>
<td>use equations and inequalities to solve real-life problems.</td>
</tr>
<tr>
<td><strong>Novice</strong></td>
</tr>
<tr>
<td>Seventh grade students at the novice level in mathematics:</td>
</tr>
<tr>
<td>simplify given expressions;</td>
</tr>
<tr>
<td>solve a given equation or inequality.</td>
</tr>
</tbody>
</table>

Use properties of operations to generate equivalent expressions.

M.7.EE.1 apply properties of operations as strategies to add, subtract, factor and expand linear expressions with rational coefficients. (CCSS Math.7.EE.1)

- Define linear expression, rational, coefficient, and rational coefficient.
- Simplify an expression by dividing by the greatest common factor (Ex. 18x + 6y = 6(3x + y)).
- Simplify expressions with parenthesis (Ex. 5(4 + x) = 20 + 5x).
- Recognize the property demonstrated in a given expression.
- Combine like terms of a given expression.
- Recall how to find the greatest common factor.
- Give examples of the properties of operations including distributive, commutative, and associative.

M.7.EE.2 understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related.

For example, a + 0.05a = 1.05a means that “increase by 5%” is the same as “multiply by 1.05.” (CCSS Math.7.EE.2)

- Define expression, equivalent, and equivalent expressions.
- Recognize that a variable without a written coefficient is understood to have a coefficient of one.
- Recall how to convert mathematical terms to mathematical symbols and numbers and vice versa.
- Restate numerical expressions with words.
- Recall mathematical terms such as sum, difference, etc.

Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

M.7.EE.3 solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. For example: If a woman making $25 an hour gets a 10% raise, she will make an additional $2.50, for a new salary of $27.50. If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation. (CCSS Math.7.EE.3)

- Define estimation, rational numbers, and reasonable.
- Analyze the given word problem to set up a mathematical problem.
- Recognize the mathematical operations of rational numbers in any form, including converting between forms. (Ex. 0.25=1/4 =25%)
- Recall problem solving methods.
- Recognize the rules of operations of positive and negative numbers.
- Recognize properties of numbers (Distributive, Associative, Commutative).
- Recall mental calculation strategies.
- Recall estimation strategies.

M.7.EE.4 use variables to represent quantities in a real-world or mathematical problem and construct simple equations and inequalities to solve problems by reasoning about the quantities.

a. Solve word problems leading to equations of the form px + q = r and p(x + q) = r, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?

b. Solve word problems leading to inequalities of the form px + q > r or px + q < r, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions.
(CCSS Math.7.EE.4)

- Define equation, inequality, and variable.
- Set up equations and inequalities to represent the given situation, using correct mathematical operations and variables.
- Calculate a solution or solution set by combining like terms, isolating the variable, and/or using inverse operations.
- Test the found number or numberset for accuracy by substitution.
- Recall solving one step equations and inequalities.
- Recognize properties of numbers (Distributive, Associative, Commutative).

a. Solve word problems leading to equations of the form px + q = r and p(x + q) = r, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach.

Example: The perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?
- Define equation and variable.
- Set up an equation to represent the given situation, using correct mathematical operations and variables.
- Calculate a solution to an equation by combining like terms, isolating the variable, and/or using inverse operations.
- Test the found number for accuracy by substitution.

Example: Is 5 an accurate solution of 2(x + 5)=12?
- Identify the unknown, in a given situation, as the variable.
- List given information from the problem.
- Recall solving one-step equations.
- Explain the distributive property.

b. Solve word problems leading to inequalities of the form px + q > r or px + q < r, where p, q, and r are specific rational numbers. Graph the solution set of the inequality, and interpret it in the context of the problem.

Example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions:
- Define inequality and variable.
- Set up an inequality to represent the given situation, using correct mathematical operations and variables.
- Calculate a solution set to an inequality by combining like terms, isolating the variable, and/or using inverse operations.
- Test the solution set for accuracy.
- Identify the unknown, of a given situation, as the variable.
- List information from the problem.
- Recall how to graph inequalities on a number line.
- Recall how to solve one step inequalities.

**Geometry**

<table>
<thead>
<tr>
<th>Performance Descriptors</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Distinguished</strong></td>
</tr>
<tr>
<td>Seventh grade students at the distinguished level in mathematics: create scale models of three-dimensional geometric figures found in nature and present in the context of a real world problem; make connections between surface area and volume in real-life situations.</td>
</tr>
</tbody>
</table>

**Draw, construct and describe geometrical figures and describe the relationships between them.**

M.7.G.1 solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale. (CCSS Math.7.G.1)

- Define scale, scale drawings, length, area, and geometric figures.
- Locate/use scale on a map.
- Identify proportional relationships.
- Recognize numeric patterns.
- Recall how to solve proportions using cross products.
M.7.G.2 draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle. (CCSS Math.7.G.2)

• Demonstrate how to use a protractor to draw an angle.
• Draw segments of a given length using a ruler.
• Recognize attributes of geometric shapes.

M.7.G.3 describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids. (CCSS Math.7.G.3)

• Define two-dimensional figure, three-dimensional figure, plane section, rectangular prism, and rectangular pyramid.
• Describe the relationship between two- and three-dimensional figures.
• Recognize symmetry.
• List attributes of three-dimensional figures.
• List attributes of two-dimensional figures.

Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.

M.7.G.4 know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle. (CCSS Math.7.G.4)

• Define circumference, area of a circle, and formula.
• Identify and label parts of a circle.
• Recognize the attributes of a circle.

M.7.G.5 use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure. (CCSS Math.7.G.5)

• Define supplementary angles, complementary angles, vertical angles, adjacent angles.
• Discuss strategies for solving multi-step problems.
• Identify all types of angles.
• Identify right angles and straight angles.
• Discuss parallel, perpendicular, and intersecting lines.

M.7.G.6 solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms. (CCSS Math.7.G.6)

• Define volume, surface area, triangles, quadrilaterals, polygons, cubes, and right prisms.
• Discuss strategies for solving real-world mathematical problems.
• Recall formulas for calculating volume and surface area.
• Identify the attributes of triangles, quadrilaterals, polygons, cubes, and right prisms.
Statistics & Probability

<table>
<thead>
<tr>
<th>Performance Descriptors</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Distinguished</strong></td>
</tr>
<tr>
<td>Seventh grade students at the distinguished level in mathematics: identify a problem, develop an hypothesis and generate a procedure to obtain and analyze data to form and present a conclusion; identify a problem, develop an hypothesis and generate a procedure to obtain and analyze data to form and present a conclusion between two populations; make and justify sound decisions based on probability.</td>
</tr>
</tbody>
</table>

Use random sampling to draw inferences about a population.

M.7.SP.1 understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences. (CCSS Math.7.SP.1)

- Define sample, validity, population, inference, random sampling, statistic, and generalization.
- Explain the validity of random sampling.
- Differentiate the appropriate sampling method.
- Analyze attributes of sample size.
- Compare sample size with population to check for validity.
- Discuss real world examples of valid sampling and generalizations.
- Recall the nature of the attribute, how it was measured, and its unit of measure.

M.7.SP.2 use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be. (CCSS Math.7.SP.2)

- Define data, random sampling, population, variation, prediction, estimation, and inference.
- Compare and contrast the random sampling data to the population.
- Predict an outcome of the entire population based on random samplings.
- Collect data from population randomly, choosing same size samples. (Ex. If population is your school, different random samplings should be same number of students)
- Recall how to calculate range, outlier, ratio, and proportion.

Draw informal comparative inferences about two populations.

M.7.SP.3 informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable. (CCSS Math.7.SP.3)

- Define measure of variability, distribution, and measure of center.
- Analyze the skew of the distributions and recognize the difference in measure of center and variability.
- Compare the measure of center and measure of variability of two distributions.
- Relate the measure of variation with the concept of ranGEO.
- Relate the measure of the center with the concept of mean.
- Recall how to calculate measure of center and measure of variability.
- Discuss how to read and interpret a graph.
M.7.SP.4 use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book. (CCSS Math.7.SP.4)

- Define measure of variability, measure of center, and inference.
- Compare the measure of center and measure of variability of two numerical data sets.
- Recall that center is related to measure of center and measure of variability is related to variation.
- Recall how to calculate measure of center and measure of variability.

Investigate chance processes and develop, use, and evaluate probability models.

M.7.SP.5 understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event. (CCSS Math.7.SP.5)

- Define probability and event.
- Recall the order of fractions on a number line.
- Demonstrate how to compare fractions with different denominators.
- Recall how to compare fractions with like denominators.

M.7.SP.6 approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times. (CCSS Math.7.SP.6)

- Define probability of chance, outcome, and event.
- Recognize the difference between possible outcomes and likely outcomes.
- Write the probability as a fraction, with likely outcomes as the numerator and possible outcomes as the denominator.
- Recall how to simplify fraction to lowest terms.
- Recognize equivalent fractions.

M.7.SP.7 develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.

a. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.

b. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies? (CCSS Math.7.SP.7)

- Define probability of chance, probability of events, outcome, and probability of observed frequency.
- Compare and contrast probability of chance and probability of observed frequency.
- Display all outcomes in a graphic representation (probability model-tree diagram, organized list, table, etc.).
- Demonstrate how to write the probability as a fraction, with likely outcomes as the numerator and possible outcomes as the denominator.
- Recall how to simplify fractions to lowest terms.
- Recognize equivalent fractions.
- Recall how to create a table or graphic display of data.
- Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. For example: If a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.

- Define probability of chance, outcome, and event.
- List all possible outcomes using a graphic representation (probability model-tree diagram, organized list, table, etc.).
- Using the model, count the frequency of the desired outcome.
- Demonstrate how to write the probability as a fraction, with likely outcomes as the numerator and possible outcomes as the denominator.
- Recall how to simplify fractions to lowest terms.
- Recognize equivalent fractions.
- Recall how to create a table or graphic display of data.
- Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. For example: Find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?

- Define probability of observed frequency, outcome, and event.
- List all actual outcome using a graphic representation (probability model-tree diagram, organized list, table, etc.).
- Using the model, count the frequency of the actual outcome.
- Demonstrate how to write the probability as a fraction, with likely outcomes as the numerator and possible outcomes as the denominator.
- Recall how to simplify fractions to lowest terms.
- Recognize equivalent fractions.
- Recall how to create a table or graphic display of data.
M.7.SP.8 find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.

a. Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.

b. Represent sample spaces for compound events using methods such as organized lists, tables, and tree diagrams. For an event described in everyday language (e.g., “rolling double sixes”), identify the outcomes in the sample space which compose the event.

c. Design and use a simulation to generate frequencies for compound events. For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?

(CCSS Math.7.SP.8)

- Define compound events, simulation, frequency, simple events, and probability of events.
- Discover when to add or multiply events to find probability of compound events.
- Choose appropriate model to display outcomes (tree diagram, organized list, or table).
- Recall how to find the probability of simple events.
- Demonstrate how to add and multiply fractions.
- Recall how to obtain a common denominator when adding fractions.
- Compute adding fractions with like denominators.
- Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.
- Define simple events and compound events.
- Discover when to add or multiply events to find probability of compound events.
- Recall how to find the probability of simple events.
- Demonstrate adding and multiplying fractions.
- Recognize how to obtain a common denominator when adding fractions.
- Represent sample spaces for compound events using methods such as organized lists, tables, and tree diagrams. For an event described in everyday language (e.g., “rolling double sixes”), identify the outcomes in the sample space which compose the event.
- Define compound events.
- Create a tree diagram including all possible outcomes.
- Choose appropriate model to display outcomes (tree diagram, organized list, or table).
- Identify the desired outcomes in model.
- Design and use a simulation to generate frequencies for compound events.
  Example: Use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?
- Define compound events, simulation, frequency, simple events, and probability of events.
- Discover when to add or multiply events to find probability of compound events.
- Recall how to find the probability of simple events.
- Demonstrate adding and multiplying fractions.
- Recognize how to obtain a common denominator when adding fractions.
- Recall how to add fractions with like denominators.
- Recall how to construct a table.
Mathematics (M) Grade 8

Content Emphases by Cluster--Grade 8*

Not all of the content in a given grade is emphasized equally in the standards. Some clusters require greater emphasis than the others based on the depth of the ideas, the time that they take to master, and/or their importance to future mathematics or the demands of college and career readiness. In addition, an intense focus on the most critical material at each grade allows depth in learning, which is carried out through the Standards for Mathematical Practice.

To say that some things have greater emphasis is not to say that anything in the standards can safely be neglected in instruction. Neglecting material will leave gaps in student skill and understanding and may leave students unprepared for the challenges of a later grade. The following table identifies the Major Clusters, Additional Clusters, and Supporting Clusters for this grade.

Key: ■ Major Clusters; □ Supporting Clusters; □ Additional Clusters

The Number System
■ Know that there are numbers that are not rational, and approximate them by rational numbers.

Expressions and Equations
■ Work with radicals and integer exponents.
■ Understand the connections between proportional relationships, lines, and linear equations.
■ Analyze and solve linear equations and pairs of simultaneous linear equations.

Functions
■ Define, evaluate, and compare functions.
■ Use functions to model relationships between quantities.

Geometry
■ Understand congruence and similarity using physical models, transparencies, or geometry software.
■ Understand and apply the Pythagorean Theorem.
□ Solve real-world and mathematical problems involving volume of cylinders, cones and spheres.

Statistics and Probability
■ Investigate patterns of association in bivariate data.

*Emphases are given at the cluster level. Refer to the Common Core State Standards for Mathematics for the specific standards that fall within each cluster.
In Grade 8, instructional time should focus on three critical areas: (1) formulating and reasoning about expressions and equations, including modeling an association in bivariate data with a linear equation, and solving linear equations and systems of linear equations; (2) grasping the concept of a function and using functions to describe quantitative relationships; (3) analyzing two- and three-dimensional space and figures using distance, angle, similarity and congruence and understanding and applying the Pythagorean Theorem.

1. Students use linear equations and systems of linear equations to represent, analyze, and solve a variety of problems. Students recognize equations for proportions (y/x = m or y = mx) as special linear equations (y = mx + b), understanding that the constant of proportionality (m) is the slope, and the graphs are lines through the origin. They understand that the slope (m) of a line is a constant rate of change, so that if the input or x-coordinate changes by an amount A, the output or y-coordinate changes by the amount m•A. Students also use a linear equation to describe the association between two quantities in bivariate data (such as arm span vs. height for students in a classroom). At this grade, fitting the model, and assessing its fit to the data are done informally. Interpreting the model in the context of the data requires students to express a relationship between the two quantities in question and to interpret components of the relationship (such as slope and y-intercept) in terms of the situation.

Students strategically choose and efficiently implement procedures to solve linear equations in one variable, understanding that when they use the properties of equality and the concept of logical equivalence, they maintain the solutions of the original equation. Students solve systems of two linear equations in two variables and relate the systems to pairs of lines in the plane; these intersect, are parallel or are the same line. Students use linear equations, systems of linear equations, linear functions and their understanding of slope of a line to analyze situations and solve problems.

2. Students grasp the concept of a function as a rule that assigns to each input exactly one output. They understand that functions describe situations where one quantity determines another. They can translate among representations and partial representations of functions (noting that tabular and graphical representations may be partial representations), and they describe how aspects of the function are reflected in the different representations.

3. Students use ideas about distance and angles, how they behave under translations, rotations, reflections and dilations and ideas about congruence and similarity to describe and analyze two-dimensional figures and to solve problems. Students show that the sum of the angles in a triangle is the angle formed by a straight line, and that various configurations of lines give rise to similar triangles because of the angles created when a transversal cuts parallel lines. Students understand the statement of the Pythagorean Theorem and its converse, and can explain why the Pythagorean Theorem holds, for example, by decomposing a square in two different ways. They apply the Pythagorean Theorem to find distances between points on the coordinate plane, to find lengths and to analyze polygons. Students complete their work on volume by solving problems involving cones, cylinders and spheres.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.
The Number System

<table>
<thead>
<tr>
<th>Performance Descriptors</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distinguished</td>
</tr>
<tr>
<td>Eighth grade students at the distinguished level in mathematics: create scenarios of real-world situations that model the use of irrational numbers.</td>
</tr>
</tbody>
</table>

Know that there are numbers that are not rational, and approximate them by rational numbers.

M.8.NS.1 know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually and convert a decimal expansion which repeats eventually into a rational number. (CCSS Math.8.NS.1)

- Define rational number and irrational number.
- Identify and give examples of rational numbers.
- Demonstrate how to convert fractions to decimals.
- Recall steps for division of fractions.

M.8.NS.2 use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram and estimate the value of expressions (e.g., \( \sqrt{2} \)). For example, by truncating the decimal expansion of \( \sqrt{2} \), show that \( \sqrt{2} \) is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations. (CCSS Math.8.NS.2)

- Define expressions and approximations.
- Identify properties of exponents.
- Recall how to compare numbers.
- Identify perfect squares and square roots.
- Demonstrate how to locate points on a vertical or horizontal number line.
- Recall how to estimate.

Expressions & Equations

<table>
<thead>
<tr>
<th>Performance Descriptors</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distinguished</td>
</tr>
<tr>
<td>Eighth grade students at the distinguished level in mathematics: within the problem situation, express numerical answers with a degree of precision appropriate for the problem context and justify reasonableness; explain the meaning of and make conjectures based on linear equations, slopes, and graphs in order to make predictions; make conjectures, collect data and investigate real-world situations leading to simultaneous linear equations; interpret the results and justify the conclusions.</td>
</tr>
</tbody>
</table>
Work with radicals and integer exponents.

M.8.EE.1 know and apply the properties of integer exponents to generate equivalent numerical expressions. For example, $32 \times 3^{-5} = 3^{-3} = 1/33 = 1/27$. (CCSS Math.8.EE.1)

- Define exponent, power, coefficient, integers, equivalent, and numerical expression.
- Restate negative exponents as positive exponents in the form $1/xy$.
- Recognize to add exponents when multiplying terms with like bases (Property of product of powers).
- Recognize to subtract exponents when dividing terms with like bases (Property of quotient of powers).
- Compute a numerical expression with positive exponents.
- Restate exponential numbers as repeated multiplication.
- Compute problems with adding and subtracting integers.

M.8.EE.2 use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where $p$ is a positive rational number.

Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational. (CCSS Math.8.EE.2)

- Define square root, cube root, inverse, perfect square, perfect cube, and irrational number.
- Recognize the inverse operation of squaring a number is square root and the inverse of cubing a number is cube root.
- Restate exponential numbers as repeated multiplication.
- Calculate the multiplication of single or multi-digit whole numbers.

M.8.EE.3 use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. For example, estimate the population of the United States as $3 \times 10^8$ and the population of the world as $7 \times 10^9$, and determine that the world population is more than 20 times larger. (CCSS Math.8.EE.3)

- Recognize a fraction as division of the denominator into the numerator.
- Recall that when dividing powers of like bases; subtract the exponents (Property of quotient of powers).
- Demonstrate how to convert fractions to a decimal.
- Recall how to write numbers in scientific notation.
- Recall estimation strategies.

M.8.EE.4 perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology. (CCSS Math.8.EE.4)

- Define scientific notation.
- Calculate multiplication and division of scientific notation.
- Recall properties of exponents.
- Discuss the real world application of scientific notation (very large or very small quantities).
- Demonstrate difference of scientific notation symbol between paper and calculator.
- Recall how to write a number using scientific notation.
- Restate exponents as repeated multiplication.

Understand the connections between proportional relationships, lines, and linear equations.

M.8.EE.5 graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed. (CCSS Math.8.EE.5)

- Define proportional relationships, unit rate, and slope.
- Demonstrate how to write ratios.
- Recall how to solve proportions using cross products.
- Demonstrate how to graph on a Cartesian plane.
- Recall how to find unit rate.

M.8.EE.6 use similar triangles to explain why the slope $m$ is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at $b$. (CCSS Math.8.EE.6)

- Define similar triangles, intercept, slope, vertical, horizontal, and origin.
- Generate the slope of a line using given ordered pairs.
- Analyze the graph to determine the rate of change.
- Demonstrate how to plot points on a coordinate plane using ordered pairs from table.
- Recall how to complete a function table.
- Recognize ordered pairs.
- Recognize similar triangles.
- Identify intersecting lines.
Analyze and solve linear equations and pairs of simultaneous linear equations.

M.8.EE.7 solve linear equations in one variable.

a. give examples of linear equations in one variable with one solution, infinitely many solutions or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where $a$ and $b$ are different numbers).

b. solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

(CCSS Math.8.EE.7)

- Define linear equation and variable.
- Recall how to solve equations for a missing variable.
- Recall properties of operation for addition and multiplication.
- Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where $a$ and $b$ are different numbers).
- Define equivalent equation.
- Recall how to solve equations by using substitution.
- Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions, using the distributive property and collecting like terms.
- Define coefficient and distributive property.
- Solve multi-step equations.
- Identify properties of operations.
- Recall how to expand expressions.
- Recall how to solve problems using the distributive property.
- Demonstrate how to simplify equations.

M.8.EE.8 analyze and solve pairs of simultaneous linear equations.

a. understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.

b. solve systems of two linear equations in two variables algebraically and estimate solutions by graphing the equations. Solve simple cases by inspection. For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6.

c. solve real-world and mathematical problems leading to two linear equations in two variables. For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.

(CCSS Math.8.EE.8)

- Define simultaneous.
- Recall how to solve linear equations.
- Recall properties of operation for addition and multiplication.
- Understand that solutions to a system of two linear equations in two variables correspond to points of intersections of their graphs because points of intersection satisfy both equations simultaneously.
- Define point of intersection.
- Recall how to solve linear equations.
- Demonstrate how to graph on the Cartesian plane.
- Identify ordered pairs.
- Recall how to solve equations by using substitution.
- Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. Example: $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6.
- Define variables.
- Recall how to estimate.
- Recall how to solve linear equations.
- Demonstrate how to graph solutions to linear equations.
- Recall how to graph ordered pairs on a Cartesian plane.
- Solve real-world and mathematical problems leading to two linear equations in two variables. Example: Given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.
- Create a word problem from given information.
- Recall how to solve linear equations.
- Explain how to write an equation to solve real-world mathematical problems.
## Functions

### Performance Descriptors

<table>
<thead>
<tr>
<th>Distinguished</th>
<th>Above Mastery</th>
<th>Mastery</th>
<th>Partial Mastery</th>
<th>Novice</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eighth grade students at the distinguished level in mathematics:</td>
<td>Eighth grade students at the above mastery level in mathematics:</td>
<td>Eighth grade students at the mastery level in mathematics:</td>
<td>Eighth grade students at the partial mastery level in mathematics:</td>
<td>Eighth grade students at the novice level in mathematics:</td>
</tr>
<tr>
<td>make conjectures about the form and meaning of functions in real-world situations and explain when a functional relationship will be linear or non-linear; make conjectures, collect data and investigate real-world situations leading to linear functions; identify the important quantities and interpret their meaning to make predictions and justify the conclusions.</td>
<td>justify the use of a certain representation of a function and fluently transform it into an alternate representation; communicate precisely the meaning of rate and initial value in $y = mx + b$ in real world situations and comfortably describe the trend of a function.</td>
<td>fluently interpret multiple representations of functions to make sense of their properties in problem situations, discern the structure and patterns of linear and non-linear functions; construct and model the relationships between quantities in linear functions, with emphasis on the rate and initial value, and communicate the qualitative relationship between the variables in functions.</td>
<td>given a functional relationship, can transform it from equation to table to a graph form; use a table to graph a line and determine rate and initial value.</td>
<td>recognize when a graph is a function; determine whether a function is increasing or decreasing.</td>
</tr>
</tbody>
</table>

### Define, evaluate, and compare functions.

M.8.F.1 understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. (function notation not required in grade 8). (CCSS Math.8.F.1)

- Define function, ordered pairs, input, output.
- Demonstrate how to plot points on a Cartesian plane using ordered pairs.
- Recall how to complete input/output tables.
- Recognize numeric patterns.

M.8.F.2 compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change. (CCSS Math.8.F.2)

- Define rate of change.
- Recognize linear equations.
- Recall how to read/interpret information from a table.
- Identify algebraic expressions.
- Recall how to name points on a Cartesian plane using ordered pairs.

M.8.F.3 interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1),(2,4) and (3,9), which are not on a straight line. (CCSS Math.8.F.3)

- Define linear and nonlinear functions.
- Recognize linear equations.
- Identify ordered pairs.

### Use functions to model relationships between quantities.

M.8.F.4 construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two $(x,y)$ values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values. (CCSS Math.8.F.4)

- Define function, rate of change, and initial value.
- Recall how to complete an input/output function table.
- Recall how to find the rate of change (slope) in a linear equation.
- Recall how to name points from a graph (ordered pairs).

M.8.F.5 describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally. (CCSS Math.8.F.5)

- Define qualitative, increase, and decrease.
- Distinguish the difference between linear and nonlinear functions.
- Recall how to plot points on a Cartesian plane.
- Identify parts of the Cartesian plane.
- Recognize ordered pairs.
Geometry

<table>
<thead>
<tr>
<th>Performance Descriptors</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Distinguished</strong></td>
</tr>
<tr>
<td>Eighth grade students at the distinguished level in mathematics: know and flexibly use the different properties of transformations to prove congruence or similarity of two-dimensional figures and of the angle relationships created when parallel lines are cut by a transversal; make conjectures and plan a pathway to a solution to solve real-world right triangle problems where the existence of triangles is not obvious; design a three-dimensional solid constructed from two or more cylinders, cones, and/or spheres with a given volume and comfortably discuss how the object meets the criteria.</td>
</tr>
</tbody>
</table>

Understand congruence and similarity using physical models, transparencies, or geometry software.

M.8.G.1 verify experimentally the properties of rotations, reflections and translations:

- lines are taken to lines, and line segments to line segments of the same length.
- angles are taken to angles of the same measure.
- parallel lines are taken to parallel lines.

(CCSS Math.8.G.1)

- Define rotation, reflection, and translation.
- Relate slides to translations.
- Relate turns to rotations.
- Relate flips to reflections.
- Lines are taken to lines, and line segments are taken to line segments of the same length.
- Distinguish between lines and line segments.
- Demonstrate how to measure length.
- Angles are taken to angles of the same measure.
- Demonstrate how to use a protractor to measure angles.
- Parallel lines are taken to parallel lines.
- Identify parallel lines.

M.8.G.2 understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections and translations; given two congruent figures, describe a sequence that exhibits the congruence between them. (CCSS Math.8.G.2)

- Define congruent and sequence.
- Recognize translations.
- Recognize reflections.
- Recognize rotations.
- Identify attributes of two-dimensional figures.
- Identify congruent figures.

M.8.G.3 describe the effect of dilations, translations, rotations and reflections on two-dimensional figures using coordinates. (CCSS Math.8.G.3)

- Define dilation.
- Recall how to find scale factor.
- Give examples of scale drawings.
- Recognize translations.
- Recognize reflections.
M.8.G.4 understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations and dilations; given two similar two dimensional figures, describe a sequence that exhibits the similarity between them. (CCSS Math.8.G.4)
  - Define similar.
  - Recognize dilations.
  - Recognize translations.
  - Recognize rotations.
  - Recognize reflections.
  - Identify similar figures.

M.8.G.5 use informal arguments to establish facts about the angle sum and exterior angle of triangles about the angles created when parallel lines are cut by a transversal and the angle-angle criterion for similarity of triangles. For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so. (CCSS Math.8.G.5)
  - Define exterior angle and transversal.
  - Identify attributes of triangles.
  - Identify supplemental angles.
  - Identify vertical angles.

Understand and apply the Pythagorean Theorem.
M.8.G.6 explain a proof of the Pythagorean Theorem and its converse. (CCSS Math.8.G.6)
  - Define Pythagorean Theorem, converse, and proof.
  - Identify right triangles.
  - Demonstrate how to find square roots.
  - Solve problems with exponents.

M.8.G.7 apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions. (CCSS Math.8.G.7)
  - Discuss strategies for solving real-world and mathematical problems.
  - Solve problems using the Pythagorean Theorem.
  - Identify right triangles.
  - Demonstrate how to find square roots.
  - Solve problems with exponents.

M.8.G.8 apply the Pythagorean Theorem to find the distance between two points in a coordinate system. (CCSS Math.8.G.8)
  - Recall how to name points on a Cartesian plane using ordered pairs.
  - Recognize ordered pairs (x, y).
  - Solve problems using the Pythagorean Theorem.
  - Identify right triangles.
  - Demonstrate how to find square roots.
  - Solve problems with exponents.

Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.
M.8.G.9 know the formulas for the volumes of cones, cylinders and spheres and use them to solve real-world and mathematical problems. (CCSS Math.8.G.9)
  - Define formula, volume, cone, cylinders, spheres, and height.
  - Discuss the measure of volume and give examples.
  - Solve problems with exponents.
  - Recall how to find circumference of a circle.
  - Identify parts of a circle.
Investigate patterns of association in bivariate data.

M.8.SP1 construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association and nonlinear association. (CCSS Math.8.SP1)

- Define bivariate scatter plot, outlier, cluster, linear, nonlinear, and positive and negative association.
- Describe patterns found in a scatter plot.
- Demonstrate how to label and plot information on a scatter plot (dot plot).
- Distinguish the difference between positive and negative correlation.
- Recall how to describe the spread of the scatter plot (dot plot).

M.8.SP2 know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line and informally assess the model fit by judging the closeness of the data points to the line. (CCSS Math.8.SP2)

- Define scatter plot, outlier, linear, quantitative, line of best fit, and variable.
- Analyze scatter plots to determine line of best fit.
- Explain how to draw informal inferences from data distributions.
- Recall how to summarize numerical data sets in relation to their context.
- Recognize the concept of outlier and its relationship to the data distribution.

M.8.SP3 use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height. (CCSS Math.8.SP3)

- Define slope, intercept, linear, equation, and bivariate.
- Recall how to determine the rate of change (slope) from a graph.
- Identify the parts of the slope-intercept form of an equation.
- Recognize how to read a graph.

M.8.SP4 understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores? (CCSS Math.8.SP4)

- Define relative frequency, bivariate, and frequency.
- Design a two-way table.
- Analyze a two-way table containing categorical variables.
- Calculate relative frequency.
- Discuss relative frequency.
- Design a table.
- Recall how to calculate frequency.
- Recall how to collect data.
Mathematics (M) High School Math I

Math 8th grade High School Math 9 does have its own set of standards. It includes content from 8th grade math as well as all of the content of Math 9. The standard can be found in Policy 2520.2B, pp 61-73.

The fundamental purpose of Mathematics I is to formalize and extend the mathematics that students learned in the middle grades. The critical areas, organized into units, deepen and extend understanding of linear relationships, in part by contrasting them with exponential phenomena, and in part by applying linear models to data that exhibit a linear trend. Mathematics I uses properties and theorems involving congruent figures to deepen and extend understanding of geometric knowledge from prior grades. The final unit in the course ties together the algebraic and geometric ideas studied. The Mathematical Practice Standards apply throughout each course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations.

Critical Area 1: By the end of eighth grade students have had a variety of experiences working with expressions and creating equations. In this first unit, students continue this work by using quantities to model and analyze situations, to interpret expressions, and by creating equations to describe situations.

Critical Area 2: In earlier grades, students define, evaluate and compare functions and use them to model relationships between quantities. In this unit, students will learn function notation and develop the concepts of domain and range. They move beyond viewing functions as processes that take inputs and yield outputs and start viewing functions as objects in their right. They explore many examples of functions, including sequences; they interpret functions given graphically, numerically, symbolically and verbally, translate between representations and understand the limitations of various representations. They work with functions given by graphs and tables, keeping in mind that, depending upon the context, these representations are likely to be approximate and incomplete. Their work includes functions that can be described or approximated by formulas as well as those that cannot. When functions describe relationships between quantities arising from a context, students reason with the units in which those quantities are measured. Students build on and informally extend their understanding of integer exponents to consider exponential functions. They compare and contrast linear and exponential functions, distinguishing between additive and multiplicative change. They interpret arithmetic sequences as linear functions and geometric sequences as exponential functions.

Critical Area 3: By the end of eighth grade, students have learned to solve linear equations in one variable and have applied graphical and algebraic methods to analyze and solve systems of linear equations in two variables. This unit builds on these earlier experiences by asking students to analyze and explain the process of solving an equation and to justify the process used in solving a system of equations. Students develop fluency writing, interpreting and translating between various forms of linear equations and inequalities and using them to solve problems. They master the solution of linear equations and apply related solution techniques and the laws of exponents to the creation and solution of simple exponential equations. Students explore systems of equations and inequalities, and they find and interpret their solutions. All of this work is grounded on understanding quantities and on relationships between them.

Critical Area 4: This unit builds upon prior students’ prior experiences with data, providing students with more formal means of assessing how a model fits data. Students use regression techniques to describe approximately linear relationships between quantities. They use graphical representations and knowledge of the context to make judgments about the appropriateness of linear models. With linear models, they look at residuals to analyze the goodness of fit.

Critical Area 5: In previous grades, students were asked to draw triangles based on given measurements. They also have prior experience with rigid motions: translations, reflections and rotations and have used these to develop notions about what it means for two objects to be congruent. Students establish triangle congruence criteria, based on analyses of rigid motions and formal constructions. They solve problems about triangles, quadrilaterals and other polygons. They apply reasoning to complete geometric constructions and explain why they work.

Critical Area 6: Building on their work with the Pythagorean Theorem in 8th grade to find distances, students use a rectangular coordinate system to verify geometric relationships, including properties of special triangles and quadrilaterals and slopes of parallel and perpendicular lines.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.
Relationships Between Quantities

### Performance Descriptors

<table>
<thead>
<tr>
<th>Distinguished</th>
<th>Above Mastery</th>
<th>Mastery</th>
<th>Partial Mastery</th>
<th>Novice</th>
</tr>
</thead>
<tbody>
<tr>
<td>Math I students at the distinguished level in mathematics: justify methods and conclusions and communicate them to others; analyze relationships within an expression to draw conclusions; justify solutions and communicate them to others; explain relationships between equations, verbal descriptions, and graphs.</td>
<td>Math I students at the above mastery level in mathematics: determine reasonableness of solution; create expressions in the context of a problem; interpret mathematical results in the context of a situation.</td>
<td>Math I students at the mastery level in mathematics: use unit analysis to determine procedures to solve problems; express numerical answers with a degree of precision appropriate for the problem context; interpret expressions in the context of a problem; analyze the relationship between quantities, recognizing constraints, in problem situations and represent them as equations and inequalities to solve problems.</td>
<td>Math I students at the partial mastery level in mathematics: select appropriate units and scale to construct a data display; select appropriate formula to solve a problem; procedurally identify terms, factors, and coefficients in a problem situation; procedurally solve a system of equations; procedurally rearrange a formula.</td>
<td>Math I students at the novice level in mathematics: create data displays when given the data and a scale; use formulas; identify terms, factors, and coefficients in an expression; solve equations and inequalities in one variable; identify the solution of a problem by reading a graph.</td>
</tr>
</tbody>
</table>

Reason quantitatively and use units to solve problems.
(Working with quantities and the relationships between them provides grounding for work with expressions, equations, and functions.)

M.1HS.RBQ.1 use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. (CCSS.Math.Content.HSN-Q.A.1)
- Interpret the scale and the origin in data displays.
- Choose the scale and the origin in graphs.
- Interpret units consistently in formulas.
- Choose units consistently in formulas.
- Use units as a way to guide the solution of multistep problems.
- Use units as a way to understand problems.
- Convert between units of measurement within the same system.

M.1HS.RBQ.2 define appropriate quantities for the purpose of descriptive modeling. (CCSS.Math.Content.HSN-Q.A.2)
- Define units of measurement.
- Identify appropriate units of measure to best describe a real-world application.

M.1HS.RBQ.3 choose a level of accuracy appropriate to limitations on measurement when reporting quantities. (CCSS.Math.Content.HSN-Q.A.3)
- Recognize the limitations for each type of measurement tool.
- Determine the level of precision needed for real-world measurements.
- Relate how rounding effects the accuracy of the measurement.

Interpret the structure of expressions.

M.1HS.RBQ.4 interpret expressions that represent a quantity in terms of its context.*
- a. interpret parts of an expression, such as terms, factors, and coefficients.
- b. interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret P(1+r)n as the product of P and a factor not depending on P. (Limit to linear expressions and to exponential expressions with integer exponents). (CCSS.Math.Content.HSA-SSE.A.1)

Create equations that describe numbers or relationships.

M.1HS.RBQ.5 create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions. (Limit to linear and exponential equations, and, in the case of exponential equations, limit to situations requiring evaluation of exponential functions at integer inputs.)/(CCSS.Math.Content.HSA-CED.A.1)
- Write equations in equivalent form to solve problem
- Solve equations for a specific variable
- Understand and apply properties of inequalities
  Example: -3x > 9 is x < -3
- Verify that a given number of variables is a solution to the equation or inequality
- Write equations to represents a real world relationship involving linear and exponential functions
Represent and solve equations and inequalities graphically.

M.1HS.LER.1 understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). (Focus on linear and exponential equations and be able to adapt and apply that learning to other types of equations in future courses.) (CCSS.Math.Content.HSA.REI.D.10)

- Understand that the graph of an equation is the solution of an equation.
- Graph a linear equation and use the graph to determine the solution set.
- Use a given graph to determine the solution set.
- Plot given points from a table.

**Linear and Exponential Relationships**

<table>
<thead>
<tr>
<th>Performance Descriptors</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Distinguished</strong></td>
</tr>
<tr>
<td>Math I students at the distinguished level in mathematics: use technological tools to explore and deepen the understanding of concepts related to representing and solving equations and inequalities; distinguish between relations that are and are not functions and communicate reasoning; use the contextual situation to make and justify predictions; use the contextual situations of two functions to make and justify predictions; use functions to draw conclusions and further analyze relationships; justify and communicate generalizations; respond to the arguments of others; find and compare the effectiveness of two plausible solution pathways; communicate carefully formulated explanations of the parameters of a function and their relationship to the solution.</td>
</tr>
</tbody>
</table>
M.1HS.LER.2 explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential and logarithmic functions.* (Focus on cases where $f(x)$ and $g(x)$ are linear or exponential.) (CCSS.Math.Content.HSA-REI.D.11)

- Define function, function notation, linear, polynomial, rational, absolute value, exponential, and logarithmic functions, and transitive property.
- Explain, using the transitive property, why the x-coordinates of the points of the graphs are solutions to the equations.
- Find solutions to the equations $y = f(x)$ and $y = g(x)$ using the graphing calculator.
- Solve equations for $y$.
- Demonstrate use of a graphing calculator, including using a table, making a graph, and finding successive approximations.

M.1HS.LER.3 graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes. (CCSS.Math.Content.HSA-REI.D.12)

- Define the half-plane as the shaded region.
- Determine the intersecting shaded region is the solution to the system.
- Graph the lines of the systems and shade the appropriate region.
- Determine whether a line should be solid or dotted, depending on the inequality symbol.
- Recognize inequality symbols $>$, $<$, $\geq$, $\leq$.

Understand the concept of a function and use function notation. (Students should experience a variety of types of situations modeled by functions. Detailed analysis of any particular class of function at this stage is not advised. Students should apply these concepts throughout their future mathematics courses. Draw examples from linear and exponential functions. In M.1HS.LER.6, draw connection to M.1HS.LER.5, which requires students to write arithmetic and geometric sequences. Emphasize arithmetic and geometric sequences as examples of linear and exponential functions.)

M.1HS.LER.4 understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If $f$ is a function and $x$ is an element of its domain, then $f(x)$ denotes the output of $f$ corresponding to the input $x$. The graph of $f$ is the graph of the equation $y = f(x)$. (CCSS.Math.Content.HSF-IF.A.1)

- Define domain, range, relation, function, table of values, input, and output.
- Understand the graph of a function is the set of ordered pairs consisting of an input and the corresponding output.
- Understand that a function is a rule that assigns to each input exactly one output.
- Identify the equation of a function, given its graph.
- Find the range of a function given its domain.
- Recognize that $f(x)$ and $y$ are the same.

M.1HS.LER.5 use function notation, evaluate functions for inputs in their domains and interpret statements that use function notation in terms of a context. (CCSS.Math.Content.HSF-IF.A.1)

- Define function notation.
- Translate a simple word problem into function notation.
- Evaluate function when given $x$-values.

M.1HS.LER.6 recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by $f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1)$ for $n = 1$. (CCSS.Math.Content.HSF-IF.A.1)

- Define sequences and recursively-defined sequences.
- Recognize that sequences are functions whose domain is the set of all positive integers and zero.

Interpret functions that arise in applications in terms of a context. M.1HS.LER.7 for a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maxima and minima; symmetries; end behavior; and periodicity. (Focus on linear and exponential functions.) (CCSS.Math.Content.HSF-IF.B.4)

- Define intercepts, intervals, relative maxima, relative minima, symmetry, end behavior, and periodicity.
- For a function that models a relationship between two quantities, find the periodicity.
- For a function that models a relationship between two quantities, find the end behavior.
- For a function that models a relationship between two quantities, find the symmetry.
- For a function that models a relationship between two quantities, find the intervals where the function is increasing, decreasing, positive, or negative.
- For a function that models a relationship between two quantities, find the relative maxima and minima.
- For a function that models a relationship between two quantities, find the $x$ and $y$ intercepts.
M.1HS.LER.8 relate the domain of a function to its graph and where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.

(Focus on linear and exponential functions.) (CCSS.Math.Content.HSF-IF.B.5)
- Define domain, range, relation, function, table of values, and mappings.
- Determine the appropriate domain for a given function.
- Identify functions from information in tables, sets of ordered pairs, and mappings.
- Translate verbal phrases into a function.
- Arrange data given as ordered pairs into a table and a table of values into ordered pairs.
- Identify the x and y values in an ordered pair

M.1HS.LER.9 calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. (Focus on linear functions and intervals for exponential functions whose domain is a subset of the integers. Mathematics II and III will address other function types.) (CCSS.Math.Content.HSF-IF.B.6)
- Define average rate of change as slope.
- Estimate the rate of change from a graph (rise/run).
- Interpret the average rate of change.
- Calculate the average rate of change.
- Compute the slope of a line given two ordered pairs.
- Identify the slope, given slope-intercept form.

Analyze functions using different representations.
(Focus on linear and exponential functions. Include comparisons of two functions presented algebraically. For example, compare the growth of two linear functions, or two exponential functions such as \(y=3^n\) and \(y=100\cdot 2^n\).)

M.1HS.LER.10 graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

a. graph linear and quadratic functions and show intercepts, maxima, and minima.

b. graph exponential and logarithmic functions, showing intercepts and end behavior and trigonometric functions, showing period, midline and amplitude.

(CCSS.Math.Content.HSF-IF.C.7)
- Define piecewise-defined functions and step functions.
- Graph functions expressed symbolically by hand in simple cases.
- Graph functions expressed symbolically using technology for more complicated cases.
- Graph quadratic functions showing maxima and minima.
- Graph quadratic functions showing intercepts.
- Graph linear functions showing intercepts
- Define square root, cube root, and absolute value function.
- Graph piecewise-defined functions.
- Graph step functions.
- Graph cube root functions.
- Graph square root functions.
- Graph absolute value functions.

M.1HS.LER.11 compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum. (CCSS.Math.Content.HSF-IF.C.9)
- Identify properties and attributes of functions.
- Compare properties and attributes of functions.
- Understand that each representation of a function (words, equation, table or graph) describes the same relationship.

Build a function that models a relationship between two quantities.
(Limit to linear and exponential functions.)

M.1HS.LER.12 write a function that describes a relationship between two quantities.

a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

b. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.

(CCSS.Math.Content.HSF-BF.A.1)
- Identify the independent and dependent variables.
- Recognize that the input is the independent variable.
- Recognize that the output is the dependent variable.
M.1HS.LER.13 write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. (Connect arithmetic sequences to linear functions and geometric sequences to exponential functions.) (CCSS.Math.Content.HSF-BF.A.2)

- Define arithmetic and geometric sequence
- Recognize that sequences are functions and sometimes defined recursively
- Recognize and extend an arithmetic sequence
- Find a given term of an arithmetic sequence (using formula \( a_n = a_1 + (n+1)d \))
- Recognize and extend geometric sequences
- Find the \( n \)th term of a geometric sequence (using formula \( a_n = a_1 \cdot r^{n-1} \))
- Show the connection between arithmetic sequence and a linear function
- Example: show both have a common difference
- Demonstrate the difference between geometric sequence and exponential function

Build new functions from existing functions.
(Focus on vertical translations of graphs of linear and exponential functions. Relate the vertical translation of a linear function to its \( y \)-intercept. While applying other transformations to a linear graph is appropriate at this level, it may be difficult for students to identify or distinguish between the effects of the other transformations included in this standard.)

M.1HS.LER.14 identify the effect on the graph of replacing \( f(x) \) by \( f(x) + k \), \( k f(x) \), \( f(kx) \), and \( f(x + k) \) for specific values of \( k \) (both positive and negative); find the value of \( k \) given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. (CCSS.Math.Content.HSF-BF.B.3)

- Introduce function notation to record the transformed function
  Example: \( g(x) = F(x-h) \) for horizontal shift
- Demonstrate ability to look at a graph or function and determine the transformation
  Example: \( f(x) = x \) and \( g(x) = x - 3 \) translates right 3 units
- Model the transformation and reflection as on a coordinate plane
- Use graphing calculators to experiment with transformations and explain the effects of the changes

Construct and compare linear, quadratic, and exponential models and solve problems.

M.1HS.LER.15 distinguish between situations that can be modeled with linear functions and with exponential functions.

a. prove that linear functions grow by equal differences over equal intervals; exponential functions grow by equal factors over equal intervals.
b. recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
c. recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. (CCSS.Math.Content.HSF-LE.A.1)

M.1HS.LER.16 construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship or two input-output pairs (include reading these from a table). (CCSS.Math.Content.HSF-LE.A.2)

- Review solving multi-step equations
- Given a set of data decide which type of function models the data and write an equation to describe it
- Look for a pattern in a data set to determine which type of model describes the data
  Example: linear has constant first difference, exponential has a constant ratio
- Identify independent and dependent variables
- Write an equation in function notation and evaluate for given input values
- Find the rate of change from a graph and write a linear equation in slope intercept form

M.1HS.LER.17 observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. (Limit to comparisons between exponential and linear models.) (CCSS.Math.Content.HSF-LE.A.3)

- Evaluate exponential functions
- Identify and graph exponential functions
- Prove that linear functions grow by equal differences over equal intervals and exponential functions grow by equal factors

Interpret expressions for functions in terms of the situation they model.
(Limit exponential functions to those of the form \( f(x) = bx + k \).)

M.1HS.LER.18 interpret the parameters in a linear or exponential function in terms of a context. (CCSS.Math.Content.HSF-LE.B.5)
Reasoning with Equations

<table>
<thead>
<tr>
<th>Performance Descriptors</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Distinguished</strong></td>
</tr>
<tr>
<td>Math I students at the distinguished level in mathematics:</td>
</tr>
<tr>
<td>analyze and evaluate alternate solution methods;</td>
</tr>
<tr>
<td>analyze and evaluate alternate solution methods;</td>
</tr>
<tr>
<td>formulate, justify, and communicate a strategy for selecting the most efficient method.</td>
</tr>
<tr>
<td><strong>Above Mastery</strong></td>
</tr>
<tr>
<td>Math I students at the above mastery level in mathematics:</td>
</tr>
<tr>
<td>communicate carefully formulated explanations of the solution and the solution pathway;</td>
</tr>
<tr>
<td>use algebraic properties to justify each step in solving inequalities in one variable and in solving literal equations;</td>
</tr>
<tr>
<td>compare the effectiveness of two plausible solution pathways.</td>
</tr>
<tr>
<td><strong>Mastery</strong></td>
</tr>
<tr>
<td>Math I students at the mastery level in mathematics:</td>
</tr>
<tr>
<td>use algebraic properties to justify each step in a simple equation;</td>
</tr>
<tr>
<td>solve and interpret solutions to inequalities in one variable; solve literal equations;</td>
</tr>
<tr>
<td>solve systems of equations, justifying that the solution pathway is mathematically valid.</td>
</tr>
<tr>
<td><strong>Partial Mastery</strong></td>
</tr>
<tr>
<td>Math I students at the partial mastery level in mathematics:</td>
</tr>
<tr>
<td>procedurally write each step to solve a simple equation;</td>
</tr>
<tr>
<td>procedurally write steps to solve inequalities in one variable;</td>
</tr>
<tr>
<td>procedurally solve systems of equations.</td>
</tr>
<tr>
<td><strong>Novice</strong></td>
</tr>
<tr>
<td>Math I students at the novice level in mathematics:</td>
</tr>
<tr>
<td>find the solution to a simple equation;</td>
</tr>
<tr>
<td>find the solution of an inequality in one variable with a positive coefficient;</td>
</tr>
<tr>
<td>demonstrate that the solution to a system satisfies both equations.</td>
</tr>
</tbody>
</table>

Understand solving equations as a process of reasoning and explain the reasoning.

(Students should focus on and master M1.RWE.1 for linear equations and be able to extend and apply their reasoning to other types of equations in future courses. Students will solve exponential equations with logarithms in Mathematics III.)

M.1HS.RWE.1 explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. (CCSS.Math.Content.HSA-REI.A.1)

- Use the properties of equality to justify each step in solving equations

Solve equations and inequalities in one variable.

(Extend earlier work with solving linear equations to solving linear inequalities in one variable and to solving literal equations that are linear in the variable being solved for. Include simple exponential equations that rely only on application of the laws of exponents, such as $5x = 125$ or $2x = 1/16$.)

M.1HS.RWE.2 solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. (CCSS.Math.Content.HSA-REI.A.2)

- Solve for different variable in formula
  Example: $d=rt$ Solve for $r$ $d/t=r$

Solve systems of equations.

(Build on student experiences graphing and solving systems of linear equations from middle school to focus on justification of the methods used. Include cases where the two equations describe the same line (yielding infinitely many solutions) and cases where two equations describe parallel lines (yielding no solution); connect to M1.CAG.2, which requires students to prove the slope criteria for parallel lines.)

M.1HS.RWE.3 prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions. (CCSS.Math.Content.HSA-REI.C.5)

- Solve a system of two linear equations by using substitution or elimination.

M.1HS.RWE.4 solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. (CCSS.Math.
Content.HSA-REI.C.6)

- Graph a system of two linear equations to see where they intersect
- The point where they intersect is the solution
### Descriptive Statistics

<table>
<thead>
<tr>
<th>Performance Descriptors</th>
<th>Distinguished</th>
<th>Above Mastery</th>
<th>Mastery</th>
<th>Partial Mastery</th>
<th>Novice</th>
</tr>
</thead>
<tbody>
<tr>
<td>Math I students at the distinguished level in mathematics:</td>
<td>Math I students at the above mastery level in mathematics:</td>
<td>Math I students at the mastery level in mathematics:</td>
<td>Math I students at the partial mastery level in mathematics:</td>
<td>Math I students at the novice level in mathematics:</td>
<td></td>
</tr>
<tr>
<td>analyze the validity of statistical summaries;</td>
<td>analyze the validity of the selection of data displays and statistical measures;</td>
<td>create single-variable data displays and identify appropriate statistical measures to compare, summarize, and interpret data;</td>
<td>create data displays for two variables and use them to recognize associations and trends; interpret linear models in the context of the data; distinguish between correlation and causation.</td>
<td>create data displays and find statistical measures; create data displays for two variables; use technology to determine the linear model and correlation coefficient.</td>
<td></td>
</tr>
<tr>
<td>analyze the validity of statistical summaries;</td>
<td>explain the interpretation of associations and trends;</td>
<td>create data displays for two variables and use them to describe associations and trends; interpret linear models in the context of the data; distinguish between correlation and causation.</td>
<td>exhibit an informal understanding of correlation coefficient.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>predict and analyze the effect of a change in the data set.</td>
<td>make conjectures concerning correlation and causation.</td>
<td>predict and analyze the effect of a change in the data set.</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Summarize, represent, and interpret data on a single count or measurement variable.**

(Students take a more sophisticated look at using a linear function to model the relationship between two numerical variables. In addition to fitting a line to data, students assess how well the model fits by analyzing residuals.)

M.1.HS.DST.1: represent data with plots on the real number line (dot plots, histograms, and box plots). (CCSS.Math.Content.HSS-ID.A.1)
- Define dot plots, histograms, and box plots.
- Represent data with plots on the real number line, using box plots.
- Represent data with plots on the real number line, using histograms.
- Represent data with plots on the real number line, using dot plots.
- Plot points using given data.

M.1.HS.DST.2: use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. (CCSS.Math.Content.HSS-ID.A.2)
- Define center, mean, median, spread, interquartile range, standard deviation, and data set.
- Calculate the interquartile range of two or more different data sets.
- Calculate the standard deviation of two or more different data sets.
- Compare the spread.
- Calculate the mean of two or more different data sets.
- Calculate the median of two or more different data sets.
- Compare the center.
- Organize data sets in either increasing or decreasing values.

M.1.HS.DST.3: interpret differences in shape, center and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). (CCSS.Math.Content.HSS-ID.A.3)
- Define outliers.
- Compare differences in shape in the context of the data sets, accounting for possible effects of outliers.
- Compare differences in center in the context of the other data sets, accounting for possible effects of outliers.
- Compare differences in spread in the context of the data sets, accounting for possible effects of outliers.

**Summarize, represent, and interpret data on two categorical and quantitative variables.**

(Students take a more sophisticated look at using a linear function to model the relationship between two numerical variables. In addition to fitting a line to data, students assess how well the model fits by analyzing residuals.)

M.1.HS.DST.4: summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data. (CCSS.Math.Content.HSS-ID.B.5)
- Define categorical data, two-way frequency table, relative frequency, joint frequency, marginal frequency, and conditional relative frequency.
- Recognize possible associations and trends in the data.
- Interpret conditional relative frequencies in the context of the data.
- Interpret marginal frequencies in the context of the data.
- Interpret joint frequencies in the context of the data.
- Interpret relative frequencies in the context of the data.
- Summarize categorical data for two categories in two-way frequency tables.
- Analyze data from tables.
M.1HS.DST.5 represent data on two quantitative variables on a scatter plot and describe how the variables are related.
   a. fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear and exponential models.
   b. informally assess the fit of a function by plotting and analyzing residuals. (Focus should be on situations for which linear models are appropriate.)
   c. fit a linear function for scatter plots that suggest a linear association.

(CCSS.Math.Content.HSS-ID.B.6)
   • Define scatter plot.
   • Describe how the variables on a scatter plot are related.
   • Represent data of two quantitative variables on a scatter plot.
   • Construct a scatter plot using given data.
   • Use functions fitted to data to solve problems in the context of the data.
   • Fit function to the data.
   • Find the equation of an exponential function given data points.
   • Find the equation of a quadratic function given data points.
   • Find the equation of a line given data points.
   • Develop a table of values from data on a graph.
   • Define fit of a function and residuals.
   • Informally assess the fit of a function by plotting one set of points from the table of values.
   • Informally assess the fit by inputting an x-value from the table into the function and comparing the results to the value on the table.
   • Define linear function and scatter plot.
   • Write the equation of a line given two points.
   • Find the slope of a line given two points.

Interpret linear models.
M.1HS.DST.6 interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. (Build on students’ work with linear relationships in eighth grade and introduce the correlation coefficient. The focus here is on the computation and interpretation of the correlation coefficient as a measure of how well the data fit the relationship.)
(CCSS.Math.Content.HSS-ID.C.7)
   • Define slope as a rate of change.
   • Understand that the y-intercept is the initial amount in the context of the data.
   • Understand that rate of change in the context of the data is the label of the y-axis divided by the label of the x-axis.

M.1HS.DST.7 compute (using technology) and interpret the correlation coefficient of a linear fit.
(CCSS.Math.Content.HSS-ID.C.8)
   • Define correlation coefficient.
   • Interpret the correlation coefficient of a linear fit.
   • Compute (using technology) the correlation coefficient of a linear fit.
   • Input data into technology to find line of best fit.

M.1HS.DST.8 distinguish between correlation and causation.
(The important distinction between a statistical relationship and a cause-and-effect relationship arises here.)
(CCSS.Math.Content.HSS-ID.C.9)
   • Define correlation and causation.
   • Distinguish between correlation and causation.

Congruence, Proof, and Constructions

<table>
<thead>
<tr>
<th>Performance Descriptors</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Distinguished</strong></td>
</tr>
<tr>
<td>Math I students at the distinguished level in mathematics: create conjectures regarding transformations and strategically use appropriate tools to test them; strategically use appropriate tools to demonstrate that SSA is not sufficient criteria for triangle congruence; identify and distinguish between correct reasoning and flawed reasoning.</td>
</tr>
</tbody>
</table>
Experiment with transformations in the plane.
(Build on student experience with rigid motions from earlier grades. Point out the basis of rigid motions in geometric concepts, e.g., translations move points a specified distance along a line parallel to a specified line; rotations move objects along a circular arc with a specified center through a specified angle.)

M.1HS.CPC.1 know precise definitions of angle, circle, perpendicular line, parallel line and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc. (CCSS.Math.Content.HSG-CO.A.1)

- Know definitions of angle, circle, perpendicular and parallel lines, and line segment.
- Understand importance of precise definitions

M.1HS.CPC.2 represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch). (CCSS.Math.Content.HSG-CO.A.2)

- Determine a new set of coordinate points based on transformations
  - Example: The point (3,9) reflected over y-axis becomes (-3,9).
- Describe the transformation given coordinate points and its image after the transformation
  - Example: Given the pre-image of (3,9) and image (3,14) know that the transformation is a translation up 5 units
- Know that some transformations (translations) maintain length and angles (rotations) while others to not (dilations).

M.1HS.CPC.3 given a rectangle, parallelogram, trapezoid or regular polygon, describe the rotations and reflections that carry it onto itself. (CCSS.Math.Content.HSG-CO.A.3)

- Describe the necessary transformations that will move a rectangle, parallelogram, trapezoid, or regular polygon on to itself (a rigid motion).
- Describe lines and points of symmetry.

M.1HS.CPC.4 develop definitions of rotations, reflections and translations in terms of angles, circles, perpendicular lines, parallel lines and line segments. (CCSS.Math.Content.HSG-CO.A.4)

- Understand the meaning of rotation, reflection and translation based on angles, circles, perpendicular and parallel lines and line segments.
- Draw rotation, reflections and translations.
- Give a polygon and its transformation identify the angle of rotation or the distance of translation,

M.1HS.CPC.5 given a geometric figure and a rotation, reflection or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another. (CCSS.Math.Content.HSG-CO.A.5)

- Transform a geometric figure given a rotation, reflection or translation using graph paper, tracing paper, or geometric software.
- Create sequences of transformations that map a figure on to itself.

Understand congruence in terms of rigid motions.
(Rigid motions are at the foundation of the definition of congruence. Students reason from the basic properties of rigid motions (that they preserve distance and angle), which are assumed without proof. Rigid motions and their assumed properties can be used to establish the usual triangle congruence criteria, which can then be used to prove other theorems.)

M.1HS.CPC.6 use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent. (CCSS.Math.Content.HSG-CO.B.6)

- Develop definition of congrueny by using transformations hat preserve size and shape.
- Justify the congruence of two figures using descriptions of rigid motions,

M.1HS.CPC.7 use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent. (CCSS.Math.Content.HSG-CO.B.7)

- Identify corresponding parts of two triangles.
- Show that two triangles are congruent if and only if corresponding parts (sides and angles) are congruent (CPCTC).

M.1HS.CPC.8 explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions. (CCSS.Math.Content.HSG-CO.B.8)

- Define congruence (based on rigid motion).
- List the minimum conditions necessary for triangle congruence (ASA, S, and SAS).
- Understand, explain, and demonstrate why SA and AAA are not sufficient to show congruence.
- Establish triangle congruence criteria using properties of rigid motion.
Make geometric constructions.
(Build on prior student experience with simple constructions. Emphasize the ability to formalize and defend how these constructions result in the desired objects. Some of these constructions are closely related to previous standards and can be introduced in conjunction with them.)

M.1HS.CPC.9 make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line. (CCSS.Math.
Content.HSG-CO.D.12)
• Have students explore how to make a variety of constructions using different tools.
• Ask students to justify how they know their method results in desired construction.
• Discuss the underlying principles that different tools rely on to produce the desired constructions.

M.1HS.CPC.10 construct an equilateral triangle, a square and a regular hexagon inscribed in a circle. (CCSS.Math.
Content.HSG-CO.D.13)

Connecting Algebra and Geometry through Coordinates

<table>
<thead>
<tr>
<th>Performance Descriptors</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Distinguished</strong></td>
</tr>
<tr>
<td>Math I students at the distinguished level in mathematics: give carefully formulated explanations showing how the distance formula is derived from the Pythagorean Theorem.</td>
</tr>
</tbody>
</table>

Use coordinates to prove simple geometric theorems algebraically.
(Reasoning with triangles in this unit is limited to right triangles; e.g., derive the equation for a line through two points using similar right triangles.)

M.1HS.CAG.1 use coordinates to prove simple geometric theorems algebraically. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, \(\sqrt{3}\)) lies on the circle centered at the origin and containing the point (0, 2). (CCSS.Math.
Content.HSG-GPE.B.4)
• Determine the slope of a line given 2 points using Slope Formula
• Determine the slope of a line given a line on the x-y coordinate plane
• Determine the length of a segment given the two endpoints using the Distance Formula
• Determine the length of a segment given endpoints of a segment on the coordinate plane by using Pythagorean Theorem
• Determine coordinates of the endpoints of a segment given endpoints of segment using Midpoint Formula
• Recall that parallel lines have the same slope
• Recall that perpendicular lines have opposite reciprocal slopes
• Recall that perpendicular segments from the right angles
• Recognize properties of special quadrilaterals (example: opposite sides of parallelogram are congruent and parallel, angles of a rectangle are right angles, etc.)

M.1HS.CAG.2 prove the slope criteria for parallel and perpendicular lines; use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point). (Relate work on parallel lines to work on M.1HS.RWE.3 involving systems of equations having no solution or infinitely many solutions.) (CCSS.Math.
Content.HSG-GPE.B.5)
• Understand the slope criteria for parallel lines
• Understand the slope criteria for perpendicular lines
• Determine an equation for a line given the slope and a point on the line
• Recognize the parallel lines have equation with no common points

M.1HS.CAG.3 use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula. (Provides practice with the distance formula and its connection with the Pythagorean theorem.) (CCSS.Math.
Content.HSG-GPE.B.7)
• Understand that the concept of perimeter of a polygon is the sum of the distances of each side
• Recall the formula for the area of a triangle \((A=1/2bh)\)
• Recall the formula for the area of a rectangle \((A=bh)\)
• Determine the length of each side of a polygon placed on the coordinate plane by using the Distance Formula and the coordinates of the endpoints of each side.
Glossary of Key Terms

Addition and subtraction within 5, 10, 20, 100, or 1000. Addition or subtraction of two whole numbers with whole number answers, and with sum or minuend in the range 0-5, 0-10, 0-20, or 0-100, respectively. Example: 8 + 2 = 10 is an addition within 10, 14 – 5 = 9 is a subtraction within 20, and 55 – 18 = 37 is a subtraction within 100.

Additive inverses. Two numbers whose sum is 0 are additive inverses of one another. Example: 3/4 and – 3/4 are additive inverses of one another because 3/4 + (– 3/4) = (– 3/4) + 3/4 = 0.

Associative property of addition. See Table 3 in this Glossary.

Associative property of multiplication. See Table 3 in this Glossary.

Bivariate data. Pairs of linked numerical observations. Example: a list of heights and weights for each player on a football team.

Box plot. A method of visually displaying a distribution of data values by using the median, quartiles, and extremes of the data set. A box shows the middle 50% of the data.

Commutative property. See Table 3 in this Glossary.

Complex fraction. A fraction A/B where A and/or B are fractions (B nonzero).

Computation algorithm. A set of predefined steps applicable to a class of problems that gives the correct result in every case when the steps are carried out correctly. See also: computation strategy.

Computation strategy. Purposeful manipulations that may be chosen for specific problems, may not have a fixed order, and may be aimed at converting one problem into another. See also: computation algorithm.

Congruent. Two plane or solid figures are congruent if one can be obtained from the other by rigid motion (a sequence of rotations, reflections, and translations).

Counting on. A strategy for finding the number of objects in a group without having to count every member of the group. For example, if a stack of books is known to have 8 books and 3 more books are added to the top, it is not necessary to count the stack all over again. One can find the total by counting on—pointing to the top book and saying “eight,” following this with “nine, ten, eleven. There are eleven books now.”

Dot plot. See: line plot.

Dilation. A transformation that moves each point along the ray through the point emanating from a fixed center, and multiplies distances from the center by a common scale factor.

Expanded form. A multi-digit number is expressed in expanded form when it is written as a sum of single-digit multiples of powers of ten. For example, 643 = 600 + 40 + 3.

Expected value. For a random variable, the weighted average of its possible values, with weights given by their respective probabilities.

First quartile. For a data set with median M, the first quartile is the median of the data values less than M. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the first quartile is 6.2 See also: median, third quartile, interquartile range.

Fraction. A number expressible in the form a/b where a is a whole number and b is a positive whole number. (The word fraction in these standards always refers to a non-negative number.) See also: rational number.

Identity property of 0. See Table 3 in this Glossary.

Independently combined probability models. Two probability models are said to be combined independently if the probability of each ordered pair in the combined model equals the product of the original probabilities of the two individual outcomes in the ordered pair.
**Integer.** A number expressible in the form $a$ or $-a$ for some whole number $a$.

**Interquartile Range.** A measure of variation in a set of numerical data, the interquartile range is the distance between the first and third quartiles of the data set. Example: For the data set $\{1, 3, 6, 7, 10, 12, 14, 15, 22, 120\}$, the interquartile range is $15 - 6 = 9$. See also: first quartile, third quartile.

**Line plot.** A method of visually displaying a distribution of data values where each data value is shown as a dot or mark above a number line. Also known as a dot plot.

**Mean.** A measure of center in a set of numerical data, computed by adding the values in a list and then dividing by the number of values in the list. Example: For the data set $\{1, 3, 6, 7, 10, 12, 14, 15, 22, 120\}$, the mean is 21.

**Mean absolute deviation.** A measure of variation in a set of numerical data, computed by adding the distances between each data value and the mean, then dividing by the number of data values. Example: For the data set $\{2, 3, 6, 7, 10, 12, 14, 15, 22, 120\}$, the mean absolute deviation is 20.

**Median.** A measure of center in a set of numerical data. The median of a list of values is the value appearing at the center of a sorted version of the list—or the mean of the two central values, if the list contains an even number of values. Example: For the data set $\{2, 3, 6, 7, 10, 12, 14, 15, 22, 90\}$, the median is 11.

**Midline.** In the graph of a trigonometric function, the horizontal line halfway between its maximum and minimum values.

**Multiplication and division within 100.** Multiplication or division of two whole numbers with whole number answers, and with product or dividend in the range 0-100. Example: $72 ÷ 8 = 9$.

**Multiplicative inverses.** Two numbers whose product is 1 are multiplicative inverses of one another. Example: $\frac{3}{4}$ and $\frac{4}{3}$ are multiplicative inverses of one another because $\frac{3}{4} \times \frac{4}{3} = \frac{4}{3} \times \frac{3}{4} = 1$.

**Number line diagram.** A diagram of the number line used to represent numbers and support reasoning about them. In a number line diagram for measurement quantities, the interval from 0 to 1 on the diagram represents the unit of measure for the quantity.

**Percent rate of change.** A rate of change expressed as a percent. Example: if a population grows from 50 to 55 in a year, it grows by $5/50 = 10\%$ per year.

**Probability distribution.** The set of possible values of a random variable with a probability assigned to each.

**Properties of operations.** See Table 3 in this Glossary.

**Properties of equality.** See Table 4 in this Glossary.

**Properties of inequality.** See Table 5 in this Glossary.

**Properties of operations.** See Table 3 in this Glossary.

**Probability.** A number between 0 and 1 used to quantify likelihood for processes that have uncertain outcomes (such as tossing a coin, selecting a person at random from a group of people, tossing a ball at a target, or testing for a medical condition).

**Probability model.** A probability model is used to assign probabilities to outcomes of a chance process by examining the nature of the process. The set of all outcomes is called the sample space, and their probabilities sum to 1. See also: uniform probability model.

**Random variable.** An assignment of a numerical value to each outcome in a sample space.

**Rational expression.** A quotient of two polynomials with a non-zero denominator.
Rational number. A number expressible in the form $a/b$ or $-a/b$ for some fraction $a/b$. The rational numbers include the integers.

Rectilinear figure. A polygon all angles of which are right angles.

Rigid motion. A transformation of points in space consisting of a sequence of one or more translations, reflections, and/or rotations. Rigid motions are here assumed to preserve distances and angle measures.

Repeating decimal. The decimal form of a rational number. See also: terminating decimal.

Sample space. In a probability model for a random process, a list of the individual outcomes that are to be considered.

Scatter plot. A graph in the coordinate plane representing a set of bivariate data. For example, the heights and weights of a group of people could be displayed on a scatter plot.

Similarity transformation. A rigid motion followed by a dilation.

Tape diagram. A drawing that looks like a segment of tape, used to illustrate number relationships. Also known as a strip diagram, bar model, fraction strip, or length model.

Terminating decimal. A decimal is called terminating if its repeating digit is 0.

Third quartile. For a data set with median $M$, the third quartile is the median of the data values greater than $M$. Example: For the data set \{2, 3, 6, 7, 10, 12, 14, 15, 22, 120\}, the third quartile is 15. See also: median, first quartile, interquartile range.

Transitivity principle for indirect measurement. If the length of object $A$ is greater than the length of object $B$, and the length of object $B$ is greater than the length of object $C$, then the length of object $A$ is greater than the length of object $C$. This principle applies to measurement of other quantities as well.

Uniform probability model. A probability model which assigns equal probability to all outcomes. See also: probability model.

Vector. A quantity with magnitude and direction in the plane or in space, defined by an ordered pair or triple of real numbers.

Visual fraction model. A tape diagram, number line diagram, or area model.

Whole numbers. The numbers 0, 1, 2, 3, ....

Resource

http://www.corestandards.org/Math/Content/mathematics-glossary/glossary