

**Report on Depth-of-Knowledge  
Distribution and Appropriateness  
of the West Virginia Mathematics  
Content Standards and Objectives  
(CSOs)**

**Prepared for the State of West Virginia**

**By**

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# **Report on Depth-of-Knowledge Distribution and Appropriateness in the West Virginia Mathematics Content Standards and Objectives (CSOs)**

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## **Introduction**

The purpose of this report is to describe an analysis of the depth-of-knowledge (DOK) levels of the K–12 West Virginia Content Standards and Objectives (CSOs), the ways in which the CSOs might be improved, and how the state’s distribution of depth of knowledge levels across grades compares to that of two other states. This analysis addressed the degree to which the distribution of DOK levels represents appropriate and challenging content for each grade level, the extent to which the content expectations become more challenging from kindergarten to grade 12, and the degree to which the DOK distribution is appropriate for each grade and across grades.

The first step in preparing the analysis was to assign an appropriate DOK level to each objective, K–12 (see Appendix A.). The DOK levels are those developed by Norman Webb and assignment is based upon experience with 21 alignment studies in 15 states, territories, and countries using his procedure.

In order to allow for comparison of DOK distribution across grade levels and states, an index number based on the average of DOK levels across a specified range of objectives was developed. The process began with assigning each objective its DOK number. After totaling these numbers at each grade level for the content domain, for example mathematics for each grade (K-12) or State A (see Section III) mathematics levels compared to West Virginia levels, this number was divided by the number of objectives, which may vary from grade to grade or from state to state. The resulting index number allowed us to make comparisons of the average complexity level, or the growth in the average complexity, across grades.

To permit analysis of the West Virginia DOK distributions, four comparisons were made. First, the distributions of objectives among the five standards were analyzed. Then, the collection of objectives for mathematics in total was analyzed. The report discusses the results and offers some explanation and evaluation of the results.

Next, the CSOs were analyzed for ways in which they might be improved. Finally, the distribution of DOKs for the West Virginia mathematics standards was compared with the distribution of complexity in the standards for two other states.

## Section I. Analysis of DOK Levels in the West Virginia CSOs

Four depth-of-knowledge (DOK) levels were used to analyze the complexity of the mathematics objectives in the West Virginia 21<sup>st</sup> Century Mathematics Content Standards and objectives. These four levels were developed to analyze the alignment between content standards and assessments and were designed to describe variation in complexity expected by the standards and required by assessment items. The lowest level describes recall or recognition of information, facts, and skills. Here a skill is considered a behavior or sequence of behaviors generally learned through practice and performed easily. The next level describes the application of skills and concepts; procedural understanding; and conceptual understanding. A concept is a class of stimuli, such as objects, events, or persons, that have common characteristics. Conceptual understanding generally refers to the integration and application of concepts and other ideas of a content area. Procedural understanding denotes knowledge about skills and sequence of steps, when and how these should be used appropriately, and their efficient and accurate applications. The third level requires more sophisticated reasoning and analysis. Assessment tasks and objectives at this level require students to solve problems; draw conclusions given data, arguments, situations, and other information; and construct mental models translating among different representations, justifying from evidence, or summarizing a body of text. The fourth level involves extended thinking that requires the integration of knowledge from multiple sources and the ability to represent knowledge in a variety of ways. Extended thinking usually requires work over a period of time, including gathering information, analyzing findings, preparing reports, and presenting findings. A more detailed description of the mathematics depth-of-knowledge levels is given in Appendix A.

A summary of the DOK levels assessed to the mathematics objectives is given in Table 1 (grades K-8) and Table 2 (high school). There is an increase in sophistication in the complexity expected by the objectives across grades. The proportion of objectives assigned a DOK level 3 (strategic thinking) generally increased with an increase in grade and level. The expectations for grades K-8 were primarily judged to expect students to engage in recall or recognition (DOK level 1) and conceptual or procedural thinking (DOK level 2). Only for grades 7 and 8 did the proportion of objectives assigned a DOK level 3 (strategic thinking) increase to over 10% of the objectives. This distribution of DOK for grades K-8 signifies a greater emphasis on skills and concepts in the lower grades with an increase in emphasis on reasoning in the middle grades. Considering only the depth-of-knowledge level, this is a reasonable distribution of complexity across the grades with the exception of grades 5 and 6. At these grades students are expected to increase their knowledge of the number system to include rational numbers (fractions, decimals, percents, and ratios) as indicated in the West Virginia CSOs. Even though, some of the objectives indicate that students are expected to analyze and solve problems involving these types of numbers, there was little indication that students are to solve other than routine problem exercises rather than more complex problems requiring students to apply these forms of numbers in non-routine ways (were students are to draw conclusions based on their own reasoning about a situation).

Table 1  
*Summary of Depth-of-Knowledge Levels for West Virginia 21<sup>st</sup> Century Mathematics Content Standards and Objectives, Grades K through 8*

| Grade | Number of objectives | DOK Level | # of objs by Level | % within std by Level |
|-------|----------------------|-----------|--------------------|-----------------------|
| K     | 27                   | 1         | 17                 | 63                    |
|       |                      | 2         | 10                 | 37                    |
|       |                      | 3         |                    |                       |
| 1     | 44                   | 1         | 30                 | 68                    |
|       |                      | 2         | 14                 | 32                    |
|       |                      | 3         |                    |                       |
| 2     | 40                   | 1         | 23                 | 58                    |
|       |                      | 2         | 14                 | 35                    |
|       |                      | 3         | 3                  | 8                     |
| 3     | 44                   | 1         | 21                 | 48                    |
|       |                      | 2         | 20                 | 45                    |
|       |                      | 3         | 3                  | 7                     |
| 4     | 46                   | 1         | 31                 | 67                    |
|       |                      | 2         | 14                 | 30                    |
|       |                      | 3         | 1                  | 3                     |
| 5     | 34                   | 1         | 16                 | 47                    |
|       |                      | 2         | 18                 | 53                    |
|       |                      | 3         |                    |                       |
| 6     | 33                   | 1         | 16                 | 48                    |
|       |                      | 2         | 15                 | 45                    |
|       |                      | 3         | 2                  | 7                     |
| 7     | 32                   | 1         | 17                 | 53                    |
|       |                      | 2         | 10                 | 31                    |
|       |                      | 3         | 5                  | 16                    |
| 8     | 30                   | 1         | 10                 | 33                    |
|       |                      | 2         | 10                 | 33                    |
|       |                      | 3         | 10                 | 33                    |

The distribution of the DOK levels for the high school courses is in general reasonable, except for the lower level courses (Algebra/Geometry Preparation, Algebra I, and Applied Mathematics I). The expectations for these courses, as judged by this one reviewer, are mainly at the recall, skill, and conceptual level. There are very few if any of the objectives for these three courses that expect students to engage in reasoning or apply their algebraic skills in non-routine problem situations. If West Virginia wants these courses to prepare students in basic skills, then the existing distribution can be appropriate. If the state wants student to be able to apply these basic skills where students have to develop a plan, think of an approach to a problem from a number of possibilities, solve the problem, and then verify the solution, then this is not communicated by the objectives for the first three high school mathematics courses. At the high school level, it

is not unreasonable for at least one-third of the objectives to be at a DOK level 3 or higher. The distribution of the DOK levels among objectives for the more advanced mathematics classes is reasonable. There are two courses—Conceptual Mathematics and Probability and Statistics—that were judged not to have any DOK 1 objectives. This suggests these courses are designed for students to build on basic skills developed in prior courses. If this is the case, then the distribution of DOK levels of these courses is reasonable.

Table 2  
*Summary of Depth-of-Knowledge Levels for West Virginia 21<sup>st</sup> Century Mathematics Content Standards and Objectives, High School*

| High School Course                         | Number of objectives | DOK Level | # of objs by Level | % within std by Level |
|--|----------------------|-----------|--------------------|-----------------------|
| Mathematics: Algebra/Geometry Preparation  | 18                   | 1         | 9                  | 50                    |
|  |                      | 2         | 9                  | 50                    |
|  |                      | 3         |                    |                       |
| Mathematics: Algebra I                     | 23                   | 1         | 9                  | 39                    |
|  |                      | 2         | 14                 | 61                    |
|  |                      | 3         |                    |                       |
| Mathematics: Applied Mathematics I         | 29                   | 1         | 11                 | 38                    |
|  |                      | 2         | 16                 | 55                    |
|  |                      | 3         | 2                  | 7                     |
| Mathematics: Geometry and Applied Geometry | 22                   | 1         | 1                  | 5                     |
|  |                      | 2         | 15                 | 68                    |
|  |                      | 3         | 6                  | 27                    |
| Mathematics: Algebra II                    | 18                   | 1         | 5                  | 28                    |
|  |                      | 2         | 11                 | 61                    |
|  |                      | 3         | 2                  | 11                    |
| Mathematics: Conceptual Mathematics        | 17                   | 1         |                    |                       |
|  |                      | 2         | 14                 | 82                    |
|  |                      | 3         | 3                  | 18                    |
| Mathematics: Trigonometry                  | 16                   | 1         | 5                  | 31                    |
|  |                      | 2         | 9                  | 56                    |
|  |                      | 3         | 2                  | 13                    |
| Mathematics: Probability and Statistics    | 14                   | 1         |                    |                       |
|  |                      | 2         | 10                 | 71                    |
|  |                      | 3         | 4                  | 29                    |
| Mathematics: Pre-calculus                  | 14                   | 1         | 1                  | 7                     |
|  |                      | 2         | 8                  | 57                    |
|  |                      | 3         | 5                  | 36                    |

None of the expectations for any grade or course was judged to require students to engage in extended thinking or reasoning (DOK level 4). In designing a K-12 mathematics curriculum, it would be reasonable that students would be expected to do a

project requiring them to apply their mathematical knowledge over an extended period of the time. This was not evident in the set of objectives analyzed here. Of course, assessing students' knowledge and skills to do such a project would be difficult to do properly on an on demand assessment. The current distribution of DOK levels across the objectives without any DOK level 4's makes it easier to develop aligned assessments to measure students' knowledge and skills.

To provide another look at the DOK level by grade/course and standard, I computed the average DOK for each standard for each grade (Table 3) and for the nine high school courses (Table 4). Again, many more factors need to be considered in judging the appropriateness of a set of objectives in addition to DOK levels. By considering the average DOK levels for each standard and grade it is possible to identify some trends and possible areas for further review.

The average DOK levels for the objectives under Standard 1 (Number and Operations) reveal that students are primarily expected to achieve recall and recognition of the number objectives (Table 3). The average objective DOK for Standard 1 for each grades, K-7, is less than 1.5. There are some number objectives that expect students to know concepts and apply skills, but most of the number objectives require students to routine operations and memorization. The average DOK level is lower for grade 5 from the other grades and should be reviewed if this is intended.

The expectations for Standard (Algebra) mainly expect students to perform at the conceptual and skill level. There is not a steady increase in average DOK across the grades, rather the average goes up and down across the grades.

Standard 3 (Geometry) is the one standard among the five that has a general increase in sophistication of expectations across the nine grades, from recall and recognition to more conceptual knowledge and skill.

Table 3  
*Average DOK Level Across Objectives by Standard and Total for Grades K-8, West Virginia 21<sup>st</sup> Century Content Standards for Mathematics*

| Grade | Number of objectives | Std 1<br>(Number)<br>Avg DOK<br>(# of Obj) | Std 2<br>(Algebra)<br>Avg DOK<br>(# of Obj) | Std 3<br>(Geometry)<br>Avg DOK<br>(# of Obj) | Std 4<br>(Measurement)<br>Avg DOK<br>(# of Obj) | Std 5<br>(Prob & Stats)<br>Avg DOK<br>(# of Obj) | Total<br>Avg<br>DOK |
|-------|----------------------|--|---|--|---|--|---------------------|
| K     | 27                   | 1.40 (10)                                  | 1.33 (3)                                    | 1.00 (4)                                     | 1.43 (7)  | 1.67 (3)   | 1.37                |
| 1     | 44                   | 1.33 (15)                                  | 1.67 (6)                                    | 1.11 (9)                                     | 1.20 (10)                                       | 1.50 (4)   | 1.32                |
| 2     | 40                   | 1.31 (13)                                  | 2.00 (5)                                    | 1.29 (7)                                     | 1.27 (11)                                       | 2.50 (4)   | 1.50                |
| 3     | 44                   | 1.44 (16)                                  | 1.83 (6)                                    | 1.38 (8)                                     | 1.70 (10)                                       | 2.00 (4)   | 1.59                |
| 4     | 46                   | 1.31 (16)                                  | 1.60 (5)                                    | 1.11 (9)                                     | 1.30 (10)                                       | 1.67 (6)   | 1.35                |
| 5     | 34                   | 1.18 (11)                                  | 1.50 (4)                                    | 1.67 (6)                                     | 1.75 (8)  | 1.80 (5)   | 1.53                |
| 6     | 33                   | 1.33 (9)                                   | 1.38 (8)                                    | 1.43 (7)                                     | 2.00 (5)  | 2.25 (4)   | 1.58                |
| 7     | 32                   | 1.33 (6)                                   | 1.83 (12)                                   | 1.67 (6)                                     | 1.20 (5)  | 2.00 (3)   | 1.63                |
| 8     | 30                   | 1.50 (4)                                   | 1.89 (9)                                    | 1.89 (9)                                     | 2.67 (3)  | 2.40 (5)   | 2.00                |

The highest average DOK level for any standard and grade is grade 8 Standard 4 (measurement). For the lower grades the measurement expectations are generally at the recall and recognition level. For grades 3, 5, and 6 the average measurement expectations are more at the conceptual knowledge and skill level. At grade 8, two of the three measurement objectives were judged to expect students to perform at the strategic thinking level.

The average probability and statistics expectations (Standard 5) is the highest across the grades for all five standards. The average is not uniform across the grades, but goes up and down from grade to grade.

The total average DOK level for all five standards does have a general increase in sophistication across the grades which is appropriate. The average DOK level for grade 4 is lower than the others and should be reviewed.

Table 4  
*Average DOK Level Across Objectives by Standard and Total for Nine High School Courses, West Virginia 21<sup>st</sup> Century Content Standards for Mathematics*

| Grade                         | Number of objectives | Std 1 (Number) Avg DOK (# of Obj) | Std 2 (Algebra) Avg DOK (# of Obj) | Std 3 (Geometry) Avg DOK (# of Obj) | Std 4 (Measurement) Avg DOK (# of Obj) | Std 5 (Prob & Stats) Avg DOK (# of Obj) | Total Avg DOK |
|-------------------------------|----------------------|-----------------------------------|------------------------------------|-------------------------------------|--|---|---------------|
| Algebra/Geometry Preparation  | 18                   | 1.00 (3)                          | 1.50 (2)                           | 1.63 (8)                            | 1.50 (2)                               | 1.67 (3)                                | 1.50          |
| Algebra I                     | 23                   |                                   | 1.61 (23)                          |                                     |  |   | 1.61          |
| Applied Mathematics I         | 29                   |                                   | 1.69 (29)                          |                                     |  |   | 1.69          |
| Geometry and Applied Geometry | 22                   |                                   |                                    | 2.23 (22)                           |  |   | 2.23          |
| Algebra II                    | 18                   |                                   | 1.83 (18)                          |                                     |  |   | 1.83          |
| Conceptual Mathematics        | 17                   |                                   | 2.17 (6)                           | 2.00 (3)                            |  | 2.25 (8)                                | 2.18          |
| Trigonometry                  | 16                   |                                   |                                    | 1.81 (16)                           |  |   | 1.81          |
| Probability and Statistics    | 14                   |                                   |                                    |                                     |  | 2.29 (14)                               | 2.29          |
|                               | 14                   |                                   | 2.27 (11)                          | 2.00 (2)                            |  | 3.00 (1)                                | 2.29          |

For the high school standards (Table 4), the average DOK level reveals a general increase in sophistication with the higher mathematics courses which is appropriate. The

Algebra/Geometry Preparation course objectives mainly expect students to recall material or reproduce work. There are few expectations that seek to have students understand the underlying concepts and apply skills. Otherwise, the high school objectives expect students to achieve a reasonable range in complexity.

## **Section II. Evaluation and Suggestions to Improve West Virginia CSOs**

The primary purpose of this analysis was to provide feedback on the depth-of-knowledge levels and not to do a completed analysis of the quality of the standards. In the process of analyzing the objective DOK levels, some observations were made on the coverage and clarity of the mathematics standards. This section discusses these observations and offers a few suggestions.

Overall, the mathematics standards across the grades and nine high school courses expect what students should know and be able to do for a basic school mathematics education. The wording of the new version of the objectives is an improvement over the former version by avoiding extensive use of verbs “to understand” and “to know.” However, some of the statements of objectives in the new version are worded more as what students should experience in instruction rather than what students should know after instruction. Verbs such as “develop”, “explore,” “discover,” “derive,” and “investigate” describe more of a process rather than an outcome that students should attain. These are more developmental stages in the acquisition of mathematical knowledge that are incorporated into the document. Some of these expectations should be included in a curriculum document rather than in a standards document that describes what students are to be held accountable for by the end of a specific grade. A copy of the standards are included in Appendix B with these works highlighted. For example, “develop fluency” with number operations states more about what students should do over instruction rather than what students will be able to do as a consequence of instruction and learning. As another example, for grade 6 measurement students are to derive a formula for the circumference of a circle. It is not clear what “derive” means. Does this mean students are to develop the formulas on their own or are they to reproduce one way a formula can be developed? If students are to show how the circumference of a circle is  $\pi D$ , this is fairly sophisticated. If they are to reproduce a derivation, then this is another outcome.

The use of “model” was difficult to determine what was meant by this word. It appears that model some times is used to indicate different forms of representations such as to write a number in expanded form. As such, model is a low level of content complexity, recall and recognition. At other times, model seems to indicate students are to use manipulatives to demonstrate conceptual understanding of an idea. In these cases, model could be a DOK level 2 or even 3. It was not clear exactly what form of model was being used. If the meaning of this word was misinterpreted, then this could change some of the assigned DOK levels.

Every grade has input/output models under Algebra. These models are one representation of a pattern and frequently are used to indicate early development of

function. The desired outcome could be stated more explicitly. At a minimum, statements of objectives should include enough specificity to distinguish what students are to do are each grade. For example, the grade 2 objective under Algebra is the same as the one for grade 1.

For some topics and grades the objectives are very specific. For example, grade 4 geometry lists objectives for shapes, angles, and lines. These all could be incorporated into one objective. Grade 5 measurement is another place where the list of objectives is too detailed and could be stated in more encompassing objectives. In general, fewer objectives makes alignment easier. However, you have to consider the trade off between clarity in communicating to your audience and efficiency for assessment purposes.

It was not clear how the grade 3 data analysis and probability objective, “analyze data represented in graphs,” differs from the grade 5 objective, “draw reasonable inferences” from a graph.

For grade 6, “plot points” in four quadrants is under algebra. However, in prior grades identifying points in a quadrant is under geometry. It is good to be consistent. A similar comment applies to 7.2.7, “use and apply scientific notation.” This objective is under algebra, but for grade 6 the same objective is under number.

At grade 4, data analysis and probability, students are to understand the uses and misuses of statistics. This is too sophisticated for this age unless I am missing something. Also, mean, median, mode, and range could all be at grade 6 rather than grades 4 and 5.

In general, the objective statements are clear and cover the appropriate content. The California mathematics content standards appear to have had some influence on the West Virginia standards as indicated by the incorporation of more rigorous algebraic outcomes in the middle grades. The middle grades objectives represent a fairly rigorous set of expectations for students.

### **Section III. Comparison of the Distribution of West Virginia’s DOK Levels Across Grades with that of Three Other States**

States vary greatly in the structure of standards. It is difficult to do a direct state-by-state comparison mainly because of how states breakdown the expectations into specific objective statements. However, when considering the coverage of the full set of standards in mathematics, state standards are very comparable.

State A is revising its mathematics standards. The information available is for the old set of standards except for grade 6. The revised standards for grade 6 written in 2005 are summarized in Table 5. State A was selected for comparison, in part, because the new standards were given a grade of B by the Fordham Foundation (Finn, Julia, and Petrilli. 2006 *The State of State Standards*, Thomas B. Fordham Foundation, August 2006, <http://www.edexcellence.net/doc/StateofStateStandards2006.pdf>). The standards summarized for the other grades are the ones written in the latter part of the 1990s. State

A has purposefully tried to increase the complexity of the standards as can be seen by the low percentage of grade 6 objectives with DOK level 1 compared to the other grades. State A revised grade 6 standards have 63 objectives. This number is larger than need be. The total number of objectives for West Virginia is more reasonable.

State B mathematics standards are very typical with an emphasis on the basic skills in the elementary grades. The recent Fordham Foundation gave State B elementary standards high remarks

. . .state's math standards are easy to read, presented in a digestible format in five simple strands. The development of arithmetic in the elementary grades is strong, with students memorizing basic number facts and calculating with whole numbers, decimals, and fractions at the appropriate ages. Standards for measurements, which tend to be problematic in other states, are direct and grade-appropriate.

The emphasis on memorizing basic facts can be seen in a majority of the State B's objectives for grades 2-7 judged to have a DOK level 1. In general, West Virginia states have a higher frequency of objectives with a DOK level 2 or 3 indicating that the content expectations have a little higher content complexity than State B. The high school standards for State A are for grade 11 high school graduation test. The high school standards for State B are a combination for grades 9 and 10. Neither state has a break down that is comparable to West Virginia.

The average DOK levels of objectives by grade are given in Table 6 for West Virginia and three other states. State C was also included to give a contrast. State C mathematics standards are clustered in fewer, but broader objectives. The average DOK level for the West Virginia objectives are fairly comparable to those for States A and B, but generally lower than those for State C. Of note, the DOK levels for West Virginia Grade 4 is lower than would be expected from the other grades for West Virginia and the other states. In general, the content complexity of the West Virginia standards reflect those expectations for a basic mathematics education that are about average compared to the other states included in this analysis.

Table 5

*Summary of Frequencies of Depth-of-Knowledge Levels of Mathematics Objectives for West Virginia and States A and B*

| Grade   | DOK Level        | Number of objectives | # of objs by Level | % within std by Level | Total number of standards | # of objs by Level  | % within std by Level | Total number of standards | # of objs by Level | % within std by Level |
|---|------------------|----------------------|--------------------|-----------------------|---------------------------|---------------------|-----------------------|---------------------------|--------------------|-----------------------|
|   |                  | West Virginia        |                    |                       | State A                   |                     |                       | State B                   |                    |                       |
| K   | 1<br>2<br>3      | 27                   | 17<br>10           | 63<br>37              |                           |                     |                       |                           |                    |                       |
| 1   | 1<br>2<br>3      | 44                   | 30<br>14           | 68<br>32              | 46                        | 26<br>19<br>1       | 56<br>41<br>2         |                           |                    |                       |
| 2   | 1<br>2<br>3      | 40                   | 23<br>14<br>3      | 58<br>35<br>8         | 45                        | 22<br>23            | 48<br>51              | 44                        | 22<br>20<br>2      | 50<br>45<br>4         |
| 3   | 1<br>2<br>3      | 44                   | 21<br>20<br>3      | 48<br>45<br>7         | 49                        | 24<br>23<br>2       | 48<br>46<br>4         | 48                        | 26<br>22           | 54<br>45              |
| 4   | 1<br>2<br>3      | 46                   | 31<br>14<br>1      | 67<br>30<br>3         | 37                        | 18<br>18<br>1       | 48<br>48<br>2         | 42                        | 22<br>17<br>3      | 52<br>40<br>7         |
| 5   | 1<br>2<br>3      | 34                   | 16<br>18           | 47<br>53              | 37                        | 18<br>18<br>1       | 48<br>48<br>2         | 50                        | 29<br>20<br>1      | 57<br>40<br>2         |
| 6   | 1<br>2<br>3<br>4 | 33                   | 16<br>15<br>2      | 48<br>45<br>7         | 63*                       | 10<br>31<br>16<br>6 | 15<br>49<br>25<br>9   | 38                        | 25<br>12<br>1      | 65<br>31<br>2         |
| 7   | 1<br>2<br>3      | 32                   | 17<br>10<br>5      | 53<br>31<br>16        | 41                        | 22<br>15<br>4       | 53<br>36<br>9         | 38                        | 11<br>22<br>4<br>1 | 28<br>57<br>10<br>2   |
| 8   | 1<br>2<br>3<br>4 | 30                   | 10<br>10<br>10     | 33<br>33<br>33        | 43                        | 24<br>16<br>3       | 55<br>37<br>6         |                           |                    |                       |
| Mathematics:<br>Algebra I                           | 1<br>2<br>3      | 23                   | 9<br>14            | 39<br>61              | 37                        | 18<br>17<br>2       | 48<br>45<br>5         |                           |                    |                       |
| Mathematics:<br>Geometry and<br>Applied<br>Geometry | 1<br>2<br>3      | 22                   | 1<br>15<br>6       | 5<br>68<br>27         | 48                        | 17<br>26<br>5       | 35<br>54<br>10        |                           |                    |                       |
| Grade 11<br>(HST Grad<br>Test)                      |                  |                      |                    |                       | 40                        | 13<br>26<br>1       | 32<br>65<br>2         | 57                        | 17<br>36<br>4      | 29<br>63<br>7         |

\* Newly written standards in 2005.

Table 6  
*Average Depth-of-Knowledge Levels for Mathematics by Grade for West Virginia and Three Other States*

| Grade                         | Number of objectives | Total Avg DOK | Number of objectives | Total Avg DOK | Number of objectives | Total Avg DOK | Number of objectives | Total Avg DOK |
|-------------------------------|----------------------|---------------|----------------------|---------------|----------------------|---------------|----------------------|---------------|
|                               | West Virginia        |               | State A              |               | State B              |               | State C              |               |
| K                             | 27                   | 1.37          |                      |               |                      |               |                      |               |
| 1                             | 44                   | 1.32          | 46                   | 1.46          |                      |               |                      |               |
| 2                             | 40                   | 1.50          | 45                   | 1.51          | 44                   | 1.54          |                      |               |
| 3                             | 44                   | 1.59          | 49                   | 1.55          | 48                   | 1.47          | 17                   | 2.00          |
| 4                             | 46                   | 1.35          | 37                   | 1.55          | 42                   | 1.55          | 17                   | 2.35          |
| 5                             | 34                   | 1.53          | 37                   | 1.61          | 50                   | 1.44          | 15                   | 2.33          |
| 6                             | 33                   | 1.58          | 63                   | 2.29*         | 38                   | 1.37          | 23                   | 1.83          |
| 7                             | 32                   | 1.63          | 41                   | 1.56          | 38                   | 1.87          | 17                   | 1.88          |
| 8                             | 30                   | 2.00          | 43                   | 1.51          |                      |               | 17                   | 1.86          |
| Algebra I                     | 23                   | 1.61          | 37                   | 1.57          |                      |               | 21                   | 1.81          |
| Geometry and Applied Geometry | 22                   | 2.23          | 48                   | 1.75          |                      |               |                      |               |
| Grade 11 (HS Grad Test)       |                      |               | 40                   | 1.68          | 57                   | 1.77          |                      |               |

# **Appendix I**

## **Definitions of Mathematics Depth-of- Knowledge Levels**

Interpreting and assigning depth-of-knowledge levels to both objectives within standards and assessment items is an essential requirement of an alignment analysis. These descriptions help to clarify what the different levels represent in mathematics:

*Level 1 (Recall)* includes the recall of information such as a fact, definition, term, or a simple procedure, as well as performing a simple algorithm or applying a formula. That is, in mathematics, a one-step, well defined, and straight algorithmic procedure should be included at this lowest level. Other key words that signify a Level 1 include “identify,” “recall,” “recognize,” “use,” and “measure.” Verbs such as “describe” and “explain” could be classified at different levels, depending on what is to be described and explained.

Level 2 (Skill/Concept) includes the engagement of some mental processing beyond a habitual response. A Level 2 assessment item requires students to make some decisions as to how to approach the problem or activity, whereas Level 1 requires students to demonstrate a rote response, perform a well-known algorithm, follow a set procedure (like a recipe), or perform a clearly defined series of steps. Keywords that generally distinguish a Level 2 item include “classify,” “organize,” “estimate,” “make observations,” “collect and display data,” and “compare data.” These actions imply more than one step. For example, to compare data requires first identifying characteristics of the objects or phenomenon and then grouping or ordering the objects. Some action verbs, such as “explain,” “describe,” or “interpret,” could be classified at different levels depending on the object of the action. For example, interpreting information from a simple graph, or requiring mathematics information from the graph, also is at Level 2. Interpreting information from a complex graph that requires some decisions on what features of the graph need to be considered and how information from the graph can be aggregated is at Level 3. Level 2 activities are not limited solely to number skills, but can involve visualization skills and probability skills. Other Level 2 activities include noticing and describing non-trivial patterns; explaining the purpose and use of experimental procedures; carrying out experimental procedures; making observations and collecting data; classifying, organizing, and comparing data; and organizing and displaying data in tables, graphs, and charts.

Level 3 (Strategic Thinking) requires reasoning, planning, using evidence, and a higher level of thinking than the previous two levels. In most instances, requiring students to explain their thinking is at Level 3. Activities that require students to make conjectures are also at this level. The cognitive demands at Level 3 are complex and abstract. The complexity does not result from the fact that there are multiple answers, a possibility for both Levels 1 and 2, but because the task requires more demanding reasoning. An activity, however, that has more than one possible answer and requires students to justify the response they give would most likely be at Level 3. Other Level 3 activities include drawing conclusions from observations; citing evidence and developing a logical argument for concepts; explaining phenomena in terms of concepts; and using concepts to solve problems.

Level 4 (Extended Thinking) requires complex reasoning, planning, developing, and thinking most likely over an extended period of time. The extended time period is not a distinguishing factor if the required work is only repetitive and does not require applying significant conceptual understanding and higher-order thinking. For example, if a student has to take the water temperature from a river each day for a month and then construct a graph, this would be classified as Level 2. However, if the student is to conduct a river study that requires taking into consideration a number of variables, this would be at Level 4. At Level 4, the cognitive demands of the task should be high and the work should be very complex. Students should be required to make several connections—relate ideas within the content area or among content areas—and to select one approach among many alternatives on how the situation should be solved, in order to be at this highest level. Level 4 activities include developing and proving conjectures; designing and conducting experiments; making connections between a finding and related concepts and phenomena; combining and synthesizing ideas into new concepts; and critiquing experimental designs.

## **Appnedix B**

**Highlighted Copy of West Virginia 21<sup>st</sup>  
Century Mathematics Content  
Standards Noting Questionable  
Wording and DOK for Each Objective**

|                   |   |                  |
|-------------------|---|------------------|
| <b>Grade K</b>    | <b>Mathematics</b>  | <b>DOK Level</b> |
| <b>Standard 1</b> | <b>Number and Operations</b>  |                  |
| M.S.K.1           | <p>Through communication, representation, reasoning and proof, problem solving, and making connections within and beyond the field of mathematics, students will</p> <ul style="list-style-type: none"> <li>• demonstrate understanding of numbers, ways of representing numbers, and relationships among numbers and number systems,</li> <li>• demonstrate meanings of operations and how they relate to one another, and</li> <li>• compute fluently and make reasonable estimates.</li> </ul> |                  |

|                   |   |   |
|-------------------|---|---|
| M.O.K.1.1         | count forward to 20 and backward from 10 with and without objects.  | 1 |
| M.O.K.1.2         | read, write, order, and compare numbers to 20.  | 1 |
| M.O.K.1.3         | group and count concrete items by ones, fives, and tens.  | 1 |
| M.O.K.1.4         | model and identify place value of each digit utilizing standard and expanded form through 20.   | 2 |
| M.O.K.1.5         | identify ordinal positions 1st – 10th and directionality.   | 1 |
| M.O.K.1.6         | estimate the number of objects in a group of 20 or less and count to determine reasonableness.  | 1 |
| M.O.K.1.7         | identify and name halves and wholes using concrete items.   | 1 |
| M.O.K.1.8         | model addition and subtraction of whole numbers using 10 or less items and write the corresponding number sentence.   | 2 |
| M.O.K.1.9         | model meanings of operations and the relationship between addition and subtraction (e.g., identity element of addition, commutative property).  | 2 |
| M.O.K.1.10        | solve grade level appropriate problems using a variety of strategies.   | 2 |
| <b>Standard 2</b> | <b>Algebra</b>  |   |
| M.S.K.2           | <p>Through communication, representation, reasoning and proof, problem solving, and making connections within and beyond the field of mathematics, students will</p> <ul style="list-style-type: none"> <li>• demonstrate understanding of patterns, relations and functions,</li> <li>• represent and analyze mathematical situations and structures using algebraic symbols,</li> <li>• use mathematical models to represent and understand quantitative relationships, and</li> <li>• analyze change in various contexts.</li> </ul> |   |

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| M.O.K.2.1         | sort and classify objects by one attribute.   | 1 |
| M.O.K.2.2         | identify, describe, and extend a repeating pattern found in common objects, sound, and movement.  | 1 |
| M.O.K.2.3         | model and identify patterns of counting by 5's and 10's.  | 2 |
| <b>Standard 3</b> | <b>Geometry</b>   |   |
| M.S.K.3           | <p><b>Through communication, representation, reasoning and proof, problem solving, and making connections within and beyond the field of mathematics, students will</b></p> <ul style="list-style-type: none"> <li>• <b>analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships,</b></li> <li>• <b>specify locations and describe spatial relationships using coordinate geometry and other representational systems,</b></li> <li>• <b>apply transformations and use symmetry to analyze mathematical situations, and</b></li> <li>• <b>solve problems using visualization, spatial reasoning, and geometric modeling.</b></li> </ul> |   |
| M.O.K.3.1         | use physical materials to construct, identify, and classify basic geometric plane shapes: circle, square, rectangle, triangle, and oval.  | 1 |
| M.O.K.3.2         | recognize basic geometric shapes in the environment.  | 1 |
| M.O.K.3.3         | model and describe spatial relationships: inside/outside, top/bottom, before/after.   | 1 |
| M.O.K.3.4         | identify the separate parts used to make a whole object.  | 1 |
| <b>Standard 4</b> | <b>Measurement</b>  |   |
| M.S.K.4           | <p>Through communication, representation, reasoning and proof, problem solving, and making connections within and beyond the field of mathematics, students will</p> <ul style="list-style-type: none"> <li>• demonstrate understanding of measurable attributes of objects and the units, systems, and processes of measurement, and</li> <li>• apply appropriate techniques, tools and formulas to determine measurements.</li> </ul>   |   |
| M.O.K.4.1         | estimate the size of an object and compare and order objects with respect to a given attribute.   | 2 |
| M.O.K.4.2         | use standard and nonstandard units of measure to find the length of an object.  | 2 |

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| M.O.K.4.3         | compare two objects in nonstandard units of measure, according to one or more of the following attributes:<br>length, height, weight.  | 2 |
| M.O.K.4.4         | name the days of the week and the seasons of the year.   | 1 |
| M.O.K.4.5         | read time to the hour using analog and digital clocks.   | 1 |
| M.O.K.4.6         | identify the name and value of a penny, nickel, and dime.  | 1 |
| M.O.K.4.7         | determine the value of a collection of pennies with a total value less than twenty cents.  | 1 |
| <b>Standard 5</b> | <b>Data Analysis and Probability</b>   |   |
| M.S.K.5           | Through communication, representation, reasoning and proof, problem solving, and making connections within and beyond the field of mathematics, students will <ul style="list-style-type: none"> <li>• formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them,</li> <li>• select and use appropriate statistical methods to analyze data,</li> <li>• develop and evaluate inferences and predictions that are based on models, and</li> <li>• apply and demonstrate an understanding of basic concepts of probability.</li> </ul> |   |
| M.O.K.5.1         | collect, sort and organize data into a graph using objects and pictures.   | 2 |
| M.O.K.5.2         | read data represented on a pictograph and a bar graph.   | 1 |
| M.O.K.5.3         | conduct a simple probability experiment and use tallies to record results in a table.  | 2 |

| <b>Grade 1</b>    | <b>Mathematics</b>   | <b>DOK Level</b> |
|-------------------|--|------------------|
| <b>Standard 1</b> | <b>Number and Operations</b>   |                  |
| M.S.1.1           | Through communication, representation, reasoning and proof, problem solving, and making connections within and beyond the field of mathematics, students will <ul style="list-style-type: none"> <li>• demonstrate understanding of numbers, ways of representing numbers, and relationships among numbers and number systems,</li> <li>• demonstrate meanings of operations and how they relate to one another, and</li> <li>• compute fluently and make reasonable estimates.</li> </ul> |                  |
| M.O.1.1.1         | count forward to 100 and backward from 20 with and without objects.  | 1                |
| M.O.1.1.2         | read, write, order, and compare numbers to 100.  | 1                |
| M.O.1.1.3         | model and identify odd and even numbers to 20 with and without objects.  | 1                |
| M.O.1.1.4         | group and count concrete items by ones and tens to 100.  | 1                |
| M.O.1.1.5         | model and identify place value of each digit utilizing standard and expanded form to 100.  | 2                |
| M.O.1.1.6         | round any two-digit number to the nearer 10 to better understand place value. *  | 1                |
| M.O.1.1.7         | identify and read ordinal numbers 1 <sup>st</sup> - 20 <sup>th</sup> .   | 1                |
| M.O.1.1.8         | estimate the number of objects in a group of 100 or less and count to evaluate reasonableness of estimate.   | 2                |
| M.O.1.1.9         | identify and name halves, thirds, and fourths as part of a whole and as part of a group using models.  | 1                |
| M.O.1.1.10        | model addition and subtraction of whole numbers using 18 or less items and write the corresponding number sentence.  | 2                |
| M.O.1.1.11        | model meanings of operations and the relationship between addition and subtraction (e.g., identity element of addition, commutative property, fact families, inverse operations).  | 2                |
| M.O.1.1.12        | demonstrate fluency with basic addition facts with sums to 10 and corresponding subtraction facts.   | 1                |
| M.O.1.1.13        | model and solve addition of three numbers with a sum of 18 or less.  | 1                |
| M.O.1.1.14        | model and solve 2-digit addition and subtraction without regrouping.   | 1                |
| M.O.1.1.15        | solve grade level appropriate picture and story problems using multiple strategies.  | 2                |
| <b>Standard 2</b> | <b>Algebra</b>   |                  |

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|-------------------|--|---|
| M.S.1.2           | Through communication, representation, reasoning and proof, problem solving, and making connections within and beyond the field of mathematics, students will <ul style="list-style-type: none"> <li>• demonstrate understanding of patterns, relations and functions,</li> <li>• represent and analyze mathematical situations and structures using algebraic symbols,</li> <li>• use mathematical models to represent and understand quantitative relationships, and</li> <li>• analyze change in various contexts.</li> </ul>   |   |
| M.O.1.2.1         | sort and classify objects by more than one attribute, using various strategies, including Venn Diagrams.   | 2 |
| M.O.1.2.2         | analyze and create a repeating pattern using common objects and numbers.   | 2 |
| M.O.1.2.3         | given an input/output model using addition or subtraction, determine the rule or give the output.  | 2 |
| M.O.1.2.4         | identify and write number patterns by 2's, 5's, and 10's.  | 1 |
| M.O.1.2.5         | identify and represent number patterns using words, AB form, and T-charts.   | 1 |
| M.O.1.2.6         | model to demonstrate that the quantities on both sides of a number sentence are equivalent.  | 2 |
| <b>Standard 3</b> | <b>Geometry</b>  |   |
| M.S.1.3           | <b>Through communication, representation, reasoning and proof, problem solving, and making connections within and beyond the field of mathematics, students will</b> <ul style="list-style-type: none"> <li>• <b>analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships,</b></li> <li>• <b>specify locations and describe spatial relationships using coordinate geometry and other representational systems,</b></li> <li>• <b>apply transformations and use symmetry to analyze mathematical situations, and</b></li> <li>• <b>solve problems using visualization, spatial reasoning, and geometric modeling.</b></li> </ul> |   |
| M.O.1.3.1         | describe and draw a triangle, square, circle, and rectangle according to number of sides and vertices.   | 1 |
| M.O.1.3.2         | use physical materials to construct, identify, and classify three-dimensional figures: cube, cone, sphere, rectangular solid, pyramid, and cylinder.   | 2 |
| M.O.1.3.3         | recognize three-dimensional shapes in the environment.   | 1 |
| M.O.1.3.4         | identify and draw open and closed figures.   | 1 |
| M.O.1.3.5         | identify and draw a line of symmetry.  | 1 |
| M.O.1.3.6         | identify and draw plane shapes that are congruent.   | 1 |

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| M.O.1.3.7         | describe spatial relationships: over/under, left/right.  | 1 |
| M.O.1.3.8         | find and name locations on a one-quadrant grid.  | 1 |
| M.O.1.3.9         | describe the shape created by combining two or more two-dimensional shapes.  | 1 |
| <b>Standard 4</b> | <b>Measurement</b>   |   |
| M.S.1.4           | Through communication, representation, reasoning and proof, problem solving, and making connections within and beyond the field of mathematics, students will <ul style="list-style-type: none"> <li>• demonstrate understanding of measurable attributes of objects and the units, systems, and processes of measurement, and</li> <li>• apply appropriate techniques, tools and formulas to determine measurements.</li> </ul> |   |
| M.O.1.4.1         | estimate, measure, compare and order using customary, metric, and nonstandard units to determine length to nearest whole unit.   | 1 |
| M.O.1.4.2         | model and compare two objects or events according to one or more of the following attributes: length, height, weight, temperature and volume, choosing the appropriate unit of measure.  | 2 |
| M.O.1.4.3         | name the months of the year and find a date on a monthly calendar.   | 1 |
| M.O.1.4.4         | explain time concept in context of personal experience.  | 2 |
| M.O.1.4.5         | read time to the half hour using an analog and digital clock.  | 1 |
| M.O.1.4.6         | calculate elapsed time to the hour.  | 1 |
| M.O.1.4.7         | identify and model the value of quarter and dollar.  | 1 |
| M.O.1.4.8         | count and trade a collection of pennies, nickels, and dimes with a total value of 100 cents or less.   | 1 |
| M.O.1.4.9         | role-play making change up to a dime.  | 1 |
| M.O.1.4.10        | select the appropriate tools of measurement to determine length, weight, volume and temperature.   | 1 |
| <b>Standard 5</b> | <b>Data Analysis and Probability</b>   |   |
| M.S.1.5           | Through communication, representation, reasoning and proof, problem solving, and making connections within and beyond the field of mathematics, students will <ul style="list-style-type: none"> <li>• formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them,</li> </ul>   |   |

|           |  |   |
|-----------|--|---|
|           | <ul style="list-style-type: none"> <li>• select and use appropriate statistical methods to analyze data,</li> <li>• develop and evaluate inferences and predictions that are based on models, and</li> <li>• apply and demonstrate an understanding of basic concepts of probability.</li> </ul> |   |
| M.O.1.5.1 | collect, sort and organize data into a bar graph.  | 2 |
| M.O.1.5.2 | interpret a pictograph with each picture representing a single unit.   | 1 |
| M.O.1.5.3 | conduct simple experiments and use the data to predict which of the events is more likely or less likely to occur if the experiment is repeated.   | 2 |
| M.O.1.5.4 | read and interpret a tally chart/table.  | 1 |

| <b>Grade 2</b>    | <b>Mathematics</b>   | <b>DOK Level</b> |
|-------------------|--|------------------|
| <b>Standard 1</b> | <b>Number and Operations</b>   |                  |
| M.S.2.1           | Through communication, representation, reasoning and proof, problem solving, and making connections within and beyond the field of mathematics, students will <ul style="list-style-type: none"> <li>• demonstrate understanding of numbers, ways of representing numbers, and relationships among numbers and number systems,</li> <li>• demonstrate meanings of operations and how they relate to one another, and</li> <li>• compute fluently and make reasonable estimates.</li> </ul> |                  |
| M.O.2.1.1         | read, write, order, and compare numbers to 1,000.  | 1                |
| M.O.2.1.2         | identify any number as odd or even.  | 1                |
| M.O.2.1.3         | count and group concrete items by 1's, 10's, and 100's to 1,000.   | 1                |
| M.O.2.1.4         | model and identify place value of each digit utilizing standard and expanded form through 1000.  | 1                |
| M.O.2.1.5         | identify and read any ordinal number.  | 1                |
| M.O.2.1.6         | round any 3-digit number to both the nearer 10 and 100.  | 1                |
| M.O.2.1.7         | identify and name fractions as part of a whole and as part of a set/group using models.  | 1                |
| M.O.2.1.8         | model and justify the relationship between addition and subtraction (e.g., identity element of addition, associative property, commutative property, inverse operations, fact families).   | 2                |
| M.O.2.1.9         | memorize basic addition facts with sums to 18 and corresponding subtraction facts.   | 1                |
| M.O.2.1.10        | model 2- and 3-digit addition and subtraction with regrouping.   | 2                |
| M.O.2.1.11        | add and subtract 2- and 3-digit numbers without regrouping.  | 1                |
| M.O.2.1.12        | use rounding to analyze the reasonableness of a sum or a difference.   | 2                |
| M.O.2.1.13        | solve story problems that require one or two-step procedures.  | 2                |
| <b>Standard 2</b> | <b>Algebra</b>   |                  |
| M.S.2.2           | Through communication, representation, reasoning and proof, problem solving, and making connections within and beyond the field of mathematics, students will <ul style="list-style-type: none"> <li>• demonstrate understanding of patterns, relations and functions,</li> </ul>  |                  |

|                   |   |   |
|-------------------|---|---|
|                   | <ul style="list-style-type: none"> <li>• represent and analyze mathematical situations and structures using algebraic symbols,</li> <li>• use mathematical models to represent and understand quantitative relationships, and</li> <li>• analyze change in various contexts.</li> </ul>   |   |
| M.O.2.2.1         | analyze, describe, extend and create a growing pattern.   | 3 |
| <b>M.O.2.2.2</b>  | <b>given an input/output model determine the rule, input or output.</b>   | 2 |
| <b>M.O.2.2.3</b>  | <b>model and identify a variety of counting patterns.</b>   | 2 |
| <b>M.O.2.2.4</b>  | <b>given the rule, complete the pattern.</b>  | 1 |
| <b>M.O.2.2.5</b>  | use models to demonstrate equivalency of two numerical expressions written as a number sentence.*   | 2 |
| <b>Standard 3</b> | <b>Geometry</b>   |   |
| M.S.2.3           | <p><b>Through communication, representation, reasoning and proof, problem solving, and making connections within and beyond the field of mathematics, students will</b></p> <ul style="list-style-type: none"> <li>• <b>analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships,</b></li> <li>• <b>specify locations and describe spatial relationships using coordinate geometry and other representational systems,</b></li> <li>• <b>apply transformations and use symmetry to analyze mathematical situations, and</b></li> <li>• <b>solve problems using visualization, spatial reasoning, and geometric modeling.</b></li> </ul> |   |
| <b>M.O.2.3.1</b>  | <b>identify and describe a cube, rectangular solid, cylinder, cone, and pyramid according to the number of faces and edges.</b>   | 1 |
| <b>M.O.2.3.2</b>  | <b>compare and contrast plane and solid geometric shapes.</b>   | 2 |
| <b>M.O.2.3.3</b>  | <b>given a design, draw the mirror image.</b>   | 2 |
| <b>M.O.2.3.4</b>  | <b>model line segments and angles.</b>  | 1 |
| <b>M.O.2.3.5</b>  | <b>identify the congruent shape that has been rotated and/or reflected.</b>   | 1 |
| <b>M.O.2.3.6</b>  | <b>plot a path between locations on a grid.</b>   | 1 |
| M.O.2.3.7         | identify similar shapes.  | 1 |

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| <b>Standard 4</b> | <b>Measurement</b>   |   |
| M.S.2.4           | Through communication, representation, reasoning and proof, problem solving, and making connections within and beyond the field of mathematics, students will <ul style="list-style-type: none"> <li>demonstrate understanding of measurable attributes of objects and the units, systems, and processes of measurement, and</li> <li>apply appropriate techniques, tools and formulas to determine measurements.</li> </ul>   |   |
| M.O.2.4.1         | use a ruler to draw and compare lengths in centimeters and inches.   | 1 |
| M.O.2.4.2         | estimate and determine the perimeter of a squares, rectangles and triangles.   | 2 |
| M.O.2.4.3         | estimate and count the number of square units needed to cover a given area.  | 2 |
| M.O.2.4.4         | estimate and determine weight/mass of familiar objects in pounds and kilograms.  | 2 |
| M.O.2.4.5         | order events in relation to time.  | 1 |
| M.O.2.4.6         | given a calendar, determine past and future days of the week and identify specific dates.  | 1 |
| M.O.2.4.7         | read time to the quarter hour using an analog and digital clock.   | 1 |
| M.O.2.4.8         | calculate elapsed time to the half hour.   | 1 |
| M.O.2.4.9         | read, write, count, and model amounts of money to a dollar.  | 1 |
| M.O.2.4.10        | model making change to a dollar.   | 1 |
| M.O.2.4.11        | read Celsius and Fahrenheit thermometers.  | 1 |
| <b>Standard 5</b> | <b><i>Data Analysis and Probability</i></b>  |   |
| M.S.2.5           | Through communication, representation, reasoning and proof, problem solving, and making connections within and beyond the field of mathematics, students will <ul style="list-style-type: none"> <li>formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them,</li> <li>select and use appropriate statistical methods to analyze data,</li> <li>develop and evaluate inferences and predictions that are based on models, and</li> <li>apply and demonstrate an understanding of basic concepts of probability.</li> </ul> |   |

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| M.O.2.5.1 | create, read, and interpret a pictograph with each picture representing greater than or equal to a single unit.   | 2 |
| M.O.2.5.2 | conduct simple experiments with more than two outcomes and use the data to predict which event is more, less, or equally likely to occur if the experiment is repeated. | 2 |
| M.O.2.5.3 | analyze data represented on a graph using grade level appropriate questions.  | 3 |
| M.O.2.5.4 | formulate questions, collect data, organize and display as a chart, table or bar graph.   | 3 |

| Grade 3    | Mathematics   | DOK Level |
|------------|---|-----------|
| Standard 1 | <i>Number and Operations</i>  |           |
| M.S.3.1    | <p>Through communication, representation, reasoning and proof, problem solving, and making connections within and beyond the field of mathematics, students will</p> <ul style="list-style-type: none"> <li>• demonstrate understanding of numbers, ways of representing numbers, and relationships among numbers and number systems,</li> <li>• demonstrate meanings of operations and how they relate to one another, and</li> <li>• compute fluently and make reasonable estimates.</li> </ul> |           |
| M.O.3.1.1  | read, write, order, and compare numbers to 10,000.  | 1         |
| M.O.3.1.2  | read, write, order, and compare decimals to hundredths, with models.  | 1         |
| M.O.3.1.3  | identify place value of each digit utilizing standard and expanded form to 10,000.  | 1         |
| M.O.3.1.4  | apply estimation skills (rounding, benchmarks, compatible numbers) to evaluate reasonableness of an answer.   | 2         |
| M.O.3.1.5  | recognize fractions as part of a whole/one and as part of a set/group using models and pictorial representations.   | 1         |
| M.O.3.1.6  | compare and order fractions with like and unlike denominators using concrete models.  | 2         |
| M.O.3.1.7  | add and subtract fractions with like denominators using concrete models and pictorial representations.  | 1         |
| M.O.3.1.8  | recognize equivalent fractions using concrete models.   | 2         |
| M.O.3.1.9  | recognize proper and improper fractions and mixed numbers using concrete models.  | 2         |
| M.O.3.1.10 | add and subtract 2- and 3-digit whole numbers and money with and without regrouping.  | 1         |
| M.O.3.1.11 | demonstrate and model multiplication (repeated addition, arrays) and division (repeated subtraction, partitioning).   | 2         |
| M.O.3.1.12 | recognize the operations of multiplication and division including the properties (e.g., identity element of multiplication, commutative property, property of zero, associative property, inverse operations).  | 1         |
| M.O.3.1.13 | develop fluency with basic multiplication facts and the corresponding division facts.*  | 1         |
| M.O.3.1.14 | model the distributive property in multiplication of 2- and 3-digit numbers by a 1-digit number.  | 1         |
| M.O.3.1.15 | use models to demonstrate division of 2- and 3-digit numbers by a 1-digit number.   | 2         |

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| M.O.3.1.16        | solve story problems involving any of the four operations using multiple strategies.   | 2 |
| <b>Standard 2</b> | <i>Algebra</i>   |   |
| M.S.3.2           | Through communication, representation, reasoning and proof, problem solving, and making connections within and beyond the field of mathematics, students will <ul style="list-style-type: none"> <li>• demonstrate understanding of patterns, relations and functions,</li> <li>• represent and analyze mathematical situations and structures using algebraic symbols,</li> <li>• use mathematical models to represent and understand quantitative relationships, and</li> <li>• analyze change in various contexts.</li> </ul>   |   |
| M.O.3.2.1         | analyze and complete patterns with geometric figures.  | 2 |
| M.O.3.2.2         | create an input/output model using addition, subtraction, multiplication or division.  | 2 |
| M.O.3.2.3         | identify, continue and write number patterns.  | 2 |
| M.O.3.2.4         | analyze a given pattern and write the rule.  | 3 |
| M.O.3.2.5         | write equivalent numerical expressions.  | 1 |
| M.O.3.2.6         | represent the idea of a variable as an unknown quantity using a symbol and determine value.  | 1 |
| <b>Standard 3</b> | <i>Geometry</i>  |   |
| M.S.3.3           | <b>Through communication, representation, reasoning and proof, problem solving, and making connections within and beyond the field of mathematics, students will</b> <ul style="list-style-type: none"> <li>• <b>analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships,</b></li> <li>• <b>specify locations and describe spatial relationships using coordinate geometry and other representational systems,</b></li> <li>• <b>apply transformations and use symmetry to analyze mathematical situations, and</b></li> <li>• <b>solve problems using visualization, spatial reasoning, and geometric modeling.</b></li> </ul> |   |
| M.O.3.3.1         | identify polygons, including triangles, through decagons.  | 1 |
| M.O.3.3.2         | identify and describe a cube, rectangular solid, cylinder, cone and pyramid according to the number of faces, edges and vertices.  | 1 |

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| M.O.3.3.3         | from a plane drawing, construct and identify the solid figure.   | 2 |
| M.O.3.3.4         | identify and draw lines of symmetry.   | 2 |
| M.O.3.3.5         | model and describe lines and rays.   | 1 |
| M.O.3.3.6         | identify and draw right, obtuse and acute angles.  | 1 |
| M.O.3.3.7         | given a model, draw an example of a flip, slide and turn (reflection, translation, and rotation).*   | 2 |
| M.O.3.3.8         | name the location of a point on a one-quadrant grid.   | 1 |
| <b>Standard 4</b> | <b>Measurement</b>   |   |
| M.S.3.4           | Through communication, representation, reasoning and proof, problem solving, and making connections within and beyond the field of mathematics, students will <ul style="list-style-type: none"> <li>demonstrate understanding of measurable attributes of objects and the units, systems, and processes of measurement, and</li> <li>apply appropriate techniques, tools and formulas to determine measurements.</li> </ul> |   |
| M.O.3.4.1         | estimate, measure, compare, order and draw lengths using inches (to the nearest ½ inch), feet, yards, centimeters and meters.  | 2 |
| M.O.3.4.2         | estimate and count the number of cubes in a rectangular solid to determine volume.   | 2 |
| M.O.3.4.3         | discover through modeling the formula for determining the area of a rectangle.*  | 3 |
| M.O.3.4.4         | estimate and measure results of mass/weight in ounces, pounds, grams, and kilograms.   | 2 |
| M.O.3.4.5         | read time to 5-minute intervals using analog and digital clocks.   | 1 |
| M.O.3.4.6         | calculate elapsed time to quarter-hour.  | 1 |
| M.O.3.4.7         | read and write amounts of money to \$100.00.   | 1 |
| M.O.3.4.8         | use coins and bills to make change up to \$10.00.  | 1 |
| M.O.3.4.9         | estimate, read, and recognize common temperatures of Celsius and Fahrenheit.   | 2 |
| M.O.3.4.10        | estimate and determine the perimeter of any polygon.   | 2 |
| <b>Standard 5</b> | <b>Data Analysis and Probability</b>   |   |
| M.S.3.5           | Through communication, representation, reasoning and proof, problem solving, and making connections within and beyond the field of mathematics, students will <ul style="list-style-type: none"> <li>formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them,</li> </ul>   |   |

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|           | <ul style="list-style-type: none"> <li>• select and use appropriate statistical methods to analyze data,</li> <li>• develop and evaluate inferences and predictions that are based on models, and</li> <li>• apply and demonstrate an understanding of basic concepts of probability.</li> </ul> |   |
| M.O.3.5.1 | collect data from observation, surveys, and experiments, and construct and label a graph.  | 2 |
| M.O.3.5.2 | use a timeline to determine a sequence of events.  | 1 |
| M.O.3.5.3 | conduct experiments to determine the likeliness of events and list all outcomes.   | 2 |
| M.O.3.5.4 | analyze data represented on a graph using grade level appropriate questions.   | 3 |

| <b>Grade 4</b>    | <b>Mathematics</b>   | <b>DOK Level</b> |
|-------------------|--|------------------|
| <b>Standard 1</b> | <b>Number and Operations</b>   |                  |
| M.S.4.1           | Through communication, representation, reasoning and proof, problem solving, and making connections within and beyond the field of mathematics, students will <ul style="list-style-type: none"> <li>• demonstrate understanding of numbers, ways of representing numbers, and relationships among numbers and number systems,</li> <li>• demonstrate meanings of operations and how they relate to one another, and</li> <li>• compute fluently and make reasonable estimates.</li> </ul> |                  |
| M.O.4.1.1         | read, write, order, and compare numbers to the millions place.   | 1                |
| M.O.4.1.2         | read, write, order, and compare decimals to thousandths with and without models and pictorial representations.   | 1                |
| M.O.4.1.3         | identify place value of each digit utilizing standard and expanded form through 1,000,000 with multiples of 10 [(5 X 10,000) + (3 X 1,000) + (4 X 10) + 2].  | 1                |
| M.O.4.1.4         | apply estimation skills (rounding, benchmarks, compatible numbers, over and under estimates) to evaluate reasonableness of an answer.  | 2                |
| M.O.4.1.5         | compare and order fractions with like and unlike denominators using benchmark fractions and the number line.   | 2                |
| M.O.4.1.6         | add and subtract fractions with like and unlike denominators using pictorial representations and concrete models.  | 1                |
| M.O.4.1.7         | recognize and model equivalent fractions using pictorial representations.  | 1                |
| M.O.4.1.8         | model both addition and subtraction of mixed numbers with and without regrouping.  | 2                |
| M.O.4.1.9         | demonstrate the relationship of fractions to decimals using concrete objects and pictorial representations.  | 1                |
| M.O.4.1.10        | round decimals to the nearest whole, 10th, or 100th place.   | 1                |
| M.O.4.1.11        | add and subtract whole numbers and decimals to the 1000th place.   | 1                |
| M.O.4.1.12        | apply the distributive property as it relates to multiplication over addition.   | 1                |
| M.O.4.1.13        | memorize basic multiplication facts and corresponding division facts.  | 1                |
| M.O.4.1.14        | divide 2-and 3-digit numbers by 1-and 2-digit numbers.   | 1                |

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| M.O.4.1.15        | solve grade level appropriate story problems using multiple strategies including simple ratios.  | 2 |
| M.4.1.16          | model and identify square, prime, and composite numbers.   | 2 |
| <b>Standard 2</b> | <b>Algebra</b>   |   |
| M.S.4.2           | Through communication, representation, reasoning and proof, problem solving, and making connections within and beyond the field of mathematics, students will <ul style="list-style-type: none"> <li>• demonstrate understanding of patterns, relations and functions,</li> <li>• represent and analyze mathematical situations and structures using algebraic symbols,</li> <li>• use mathematical models to represent and understand quantitative relationships, and</li> <li>• analyze change in various contexts.</li> </ul>   |   |
| M.O.4.2.1         | given an input/output model using two operations, determine the rule.  | 2 |
| M.O.4.2.2         | recognize and describe a relationship in which quantities change proportionally.   | 2 |
| M.O.4.2.3         | represent the idea of a variable as an unknown quantity using a letter.  | 1 |
| M.O.4.2.4         | write an expression using a variable to describe a situation.  | 2 |
| M.O.4.2.5         | incorporate the order of operations (parentheses and four operations) in solving problems and writing expressions.   | 1 |
| <b>Standard 3</b> | <b>Geometry</b>  |   |
| M.S.4.3           | <b>Through communication, representation, reasoning and proof, problem solving, and making connections within and beyond the field of mathematics, students will</b> <ul style="list-style-type: none"> <li>• <b>analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships,</b></li> <li>• <b>specify locations and describe spatial relationships using coordinate geometry and other representational systems,</b></li> <li>• <b>apply transformations and use symmetry to analyze mathematical situations, and</b></li> <li>• <b>solve problems using visualization, spatial reasoning, and geometric modeling.</b></li> </ul> |   |
| M.O.4.3.1         | identify and classify plane figures and their components.  | 1 |
| M.O.4.3.2         | compare and contrast quadrilateral shapes.   | 1 |
| M.O.4.3.3         | recognize and describe three-dimensional objects from different perspectives.  | 1 |

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| M.O.4.3.4         | identify and draw intersecting, parallel, and perpendicular lines.   | 1 |
| M.O.4.3.5         | draw, label, compare, and classify acute, right, obtuse and straight angles.   | 1 |
| M.O.4.3.6         | create a design with one line of symmetry.   | 2 |
| M.O.4.3.7         | graph/plot ordered pairs on a one-quadrant grid and use the coordinate system to specify location and describe path.   | 1 |
| M.O.4.3.8         | draw and identify parts of a circle: center point, diameter, and radius.   | 1 |
| M.O.4.3.9         | identify the images of figures after reflections, translations and rotations.  | 1 |
| <b>Standard 4</b> | <b>Measurement</b>   |   |
| M.S.4.4           | Through communication, representation, reasoning and proof, problem solving, and making connections within and beyond the field of mathematics, students will <ul style="list-style-type: none"> <li>• demonstrate understanding of measurable attributes of objects and the units, systems, and processes of measurement, and</li> <li>• apply appropriate techniques, tools and formulas to determine measurements.</li> </ul> |   |
| M.O.4.4.1         | estimate, measure, compare, order and draw lengths using customary (to the nearest one-fourth inch) and metric units.  | 2 |
| M.O.4.4.2         | determine and compare areas of rectangles and squares by multiplying length and width.   | 1 |
| M.O.4.4.3         | discover through modeling the formula for volume of a rectangular prism.*  | 2 |
| M.O.4.4.4         | demonstrate conversions within a system of measure, in relation to capacity and volume.  | 1 |
| M.O.4.4.5         | read scales of weight, capacity, and temperature and select appropriate unit.  | 1 |
| M.O.4.4.6         | read time to the minute.   | 1 |
| M.O.4.4.7         | determine elapsed time in hours/ minutes within a 24-hour period.  | 1 |
| M.O.4.4.8         | count coins and bills and determine correct change.  | 1 |
| M.O.4.4.9         | given the perimeter of a polygon, determine the missing length of a side.  | 2 |
| M.O.4.4.10        | select appropriate customary and metric units and the tools for measuring to desired degree of precision.  | 1 |
| <b>Standard 5</b> | <b>Data Analysis and Probability</b>   |   |
| M.S.4.5           | Through communication, representation, reasoning and proof, problem solving, and making connections  |   |

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|           | <p>within and beyond the field of mathematics, students will</p> <ul style="list-style-type: none"> <li>• formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them,</li> <li>• select and use appropriate statistical methods to analyze data,</li> <li>• develop and evaluate inferences and predictions that are based on models, and</li> <li>• apply and demonstrate an understanding of basic concepts of probability.</li> </ul> |   |
| M.O.4.5.1 | understand and reason about the use and misuse of statistics in our society.  | 3 |
| M.O.4.5.2 | read and interpret information represented on a circle graph.   | 2 |
| M.O.4.5.3 | collect, organize, read, display and interpret data and compare sets of data from a problem solving situation.  | 2 |
| M.O.4.5.4 | examine and list all possible combinations in a given situation using a tree diagram.   | 1 |
| M.O.4.5.5 | identify and calculate median, mode and range from collected data.  | 1 |
| M.O.4.5.6 | represent the outcomes of a simple experiment as a ratio.   | 1 |

| <b>Grade 5</b>    | <b>Mathematics</b>   | <b>DOK Level</b> |
|-------------------|--|------------------|
| <b>Standard 1</b> | <b>Number and Operations</b>   |                  |
| M.S.5.1           | Through communication, representation, reasoning and proof, problem solving, and making connections within and beyond the field of mathematics, students will <ul style="list-style-type: none"> <li>• demonstrate understanding of numbers, ways of representing numbers, and relationships among numbers and number systems,</li> <li>• demonstrate meanings of operations and how they relate to one another, and</li> <li>• compute fluently and make reasonable estimates.</li> </ul> |                  |
| M.O.5.1.1         | read, write, order and compare all whole numbers, fractions, mixed numbers and decimals.   | 1                |
| M.O.5.1.2         | identify place value of each digit utilizing standard and expanded form in any whole number using powers of 10 $[(3 \times 10^5) + (4 \times 10^3) + 7 \times 10^2) + (1 \times 10^1) + 6]$ .  | 1                |
| M.O.5.1.3         | estimate solutions to problems involving whole numbers, decimals, fractions, and percents to determine reasonableness using benchmarks.  | 2                |
| M.O.5.1.4         | identify and use the divisibility rules of 2, 3, 5, 9 and 10.  | 1                |
| M.O.5.1.5         | Identify greatest common factor and lowest common multiple of two numbers.   | 1                |
| M.O.5.1.6         | model and write equivalencies of fractions, decimals, percents, and ratios.  | 1                |
| M.O.5.1.7         | add and subtract fractions and mixed numbers.  | 1                |
| M.O.5.1.8         | model and calculate multiplication and division of fractions and mixed numbers.  | 1                |
| M.O.5.1.9         | model and calculate multiplication of decimals and division of decimals by a whole number divisor.   | 1                |
| M.O.5.1.10        | demonstrate fluency in addition, subtraction, multiplication and division of whole numbers.  | 1                |
| M.O.5.1.11        | solve word problems involving whole numbers, decimals and fractions using multiple strategies.   | 2                |
| <b>Standard 2</b> | <b>Algebra</b>   |                  |
| M.S.5.2           | Through communication, representation, reasoning and proof, problem solving, and making connections within and beyond the field of mathematics, students will <ul style="list-style-type: none"> <li>• demonstrate understanding of patterns, relations and functions,</li> <li>• represent and analyze mathematical situations and structures using algebraic symbols,</li> <li>• use mathematical models to represent and understand quantitative relationships, and</li> </ul>          |                  |

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|                   | <ul style="list-style-type: none"> <li>analyze change in various contexts.</li> </ul>   |   |
| M.O.5.2.1         | explore and identify a variety of patterns with missing elements (e.g., square numbers, arithmetic sequences).  | 1 |
| M.O.5.2.2         | given an input/output model using two operations, determine the rule, output or input.  | 2 |
| M.O.5.2.3         | model, then write an expression using a variable to solve problems.   | 2 |
| M.O.5.2.4         | <b>evaluate an expression given a value for the variable.</b>   | 1 |
| <b>Standard 3</b> | <b>Geometry</b>   |   |
| M.S.5.3           | <p><b>Through communication, representation, reasoning and proof, problem solving, and making connections within and beyond the field of mathematics, students will</b></p> <ul style="list-style-type: none"> <li><b>analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships,</b></li> <li><b>specify locations and describe spatial relationships using coordinate geometry and other representational systems,</b></li> <li><b>apply transformations and use symmetry to analyze mathematical situations, and</b></li> <li><b>solve problems using visualization, spatial reasoning, and geometric modeling.</b></li> </ul> |   |
| M.O.5.3.1         | classify and compare triangles by sides and angles.   | 1 |
| M.O.5.3.2         | construct a 3-dimensional figure from different views (orthogonal drawings).  | 2 |
| M.O.5.3.3         | measure angles using a protractor.  | 1 |
| M.O.5.3.4         | create a design with more than one line of symmetry.  | 2 |
| M.O.5.3.5         | construct a circle with a given radius or diameter.   | 2 |
| M.O.5.3.6         | draw a similar figure using a scale.  | 2 |
| <b>Standard 4</b> | <b>Measurement</b>  |   |
| M.S.5.4           | <p>Through communication, representation, reasoning and proof, problem solving, and making connections within and beyond the field of mathematics, students will</p> <ul style="list-style-type: none"> <li>demonstrate understanding of measurable attributes of objects and the units, systems, and processes of measurement, and</li> </ul>  |   |

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|                   | <ul style="list-style-type: none"> <li>• apply appropriate techniques, tools and formulas to determine measurements.</li> </ul>   |   |
| M.O.5.4.1         | estimate, measure, compare, order and draw lengths of real objects in parts of an inch up to 1/8 of an inch and millimeters.  | 2 |
| M.O.5.4.2         | model, calculate and compare area of triangles and parallelograms using appropriate formulas.*  | 2 |
| M.O.5.4.3         | solve problems using the formula for determining volume of a rectangular prism.   | 2 |
| M.O.5.4.4         | investigate and describe the relationship between perimeter and area of polygons.*  | 2 |
| M.O.5.4.5         | solve application problems requiring conversions within a system of measurement.  | 2 |
| M.O.5.4.6         | evaluate and/or measure the weight/mass of real objects in ounces, pounds, grams, and kilograms.  | 1 |
| M.O.5.4.7         | calculate elapsed time to solve problems.   | 1 |
| M.O.5.4.8         | calculate actual measurement from scale drawings.   | 2 |
| <b>Standard 5</b> | <b>Data Analysis and Probability</b>  |   |
| M.S.5.5           | <p>Through communication, representation, reasoning and proof, problem solving, and making connections within and beyond the field of mathematics, students will</p> <ul style="list-style-type: none"> <li>• formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them,</li> <li>• select and use appropriate statistical methods to analyze data,</li> <li>• develop and evaluate inferences and predictions that are based on models, and</li> <li>• apply and demonstrate an understanding of basic concepts of probability.</li> </ul> |   |
| M.O.5.5.1         | collect, organize, display, read and interpret data from a problem-solving situation in a stem and leaf plot.   | 2 |
| M.O.5.5.2         | solve problems involving the probability of an event by construction of a sample space representing all possible results.   | 2 |
| M.O.5.5.3         | construct, read, and interpret tables, charts, and graphs to draw reasonable inferences or verify predictions.  | 2 |
| M.O.5.5.4         | construct a circle graph from given data.   | 2 |
| M.O.5.5.5         | calculate and compare mean, median, mode and range from collected data.   | 1 |

| Grade 6           | Mathematics   | DOK Level |
|-------------------|---|-----------|
| <b>Standard 1</b> | <b>Number and Operations</b>  |           |
| M.S.6.1           | <p>Through communication, representation, reasoning and proof, problem solving, and making connections within and beyond the field of mathematics, students will</p> <ul style="list-style-type: none"> <li>• demonstrate understanding of numbers, ways of representing numbers, and relationships among numbers and number systems,</li> <li>• demonstrate meanings of operations and how they relate to one another, and</li> <li>• compute fluently and make reasonable estimates.</li> </ul>                                       |           |
| M.O.6.1.1         | read, write, order, and compare numbers using scientific notation.  | 1         |
| M.O.6.1.2         | use prime factorization to determine the greatest common factor and least common multiple.  | 1         |
| M.O.6.1.3         | represent integers on a number line.  | 1         |
| M.O.6.1.4         | analyze and solve problems in context involving addition, subtraction, multiplication and division of whole numbers, fractions, mixed numbers and decimals and justify the reasonableness by estimation.  | 3         |
| M.O.6.1.5         | identify, demonstrate and apply the distributive, commutative and associative and identify properties.  | 1         |
| M.O.6.1.6         | convert between fractions, mixed numbers, decimals and percents.  | 1         |
| M.O.6.1.7         | compute the percent of a number to solve problems.  | 1         |
| M.O.6.1.8         | interpret the effect of multiplying and dividing rational numbers, whole numbers, fractions and decimals by numbers equal to 0 and 1 and between 0 and 1.   | 1         |
| M.O.6.1.9         | model and develop the rules for addition, subtraction, multiplication and division of integers. *   | 2         |
| <b>Standard 2</b> | <i>Algebra</i>  |           |
| M.S.6.2           | <p>Through communication, representation, reasoning and proof, problem solving, and making connections within and beyond the field of mathematics, students will</p> <ul style="list-style-type: none"> <li>• demonstrate understanding of patterns, relations and functions,</li> <li>• represent and analyze mathematical situations and structures using algebraic symbols,</li> <li>• use mathematical models to represent and understand quantitative relationships, and</li> <li>• analyze change in various contexts.</li> </ul> |           |

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| M.O.6.2.1         | simplify algebraic and numerical expressions using order of operations.   | 1 |
| M.O.6.2.2         | identify and combine like terms and monomials.  | 1 |
| M.O.6.2.3         | explore and extend patterns to predict the nth term (e.g., powers and triangular numbers).  | 2 |
| M.O.6.2.4         | translate word phrases into algebraic expressions to solve problems.  | 1 |
| M.O.6.2.5         | use input/output models and spreadsheets to evaluate functions.   | 2 |
| M.O.6.2.6         | use cross multiplication to solve proportion problems involving rates, statistics and measurements.   | 1 |
| M.O.6.2.7         | locate and plot points within the four quadrants.   | 1 |
| M.O.6.2.8         | use linear equations to represent and solve real world problems.  | 2 |
| <b>Standard 3</b> | <b>Geometry</b>   |   |
| M.S.6.3           | <p style="text-align: center;"><b>Through communication, representation, reasoning and proof, problem solving, and making connections within and beyond the field of mathematics, students will</b></p> <ul style="list-style-type: none"> <li>• <b>analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships,</b></li> <li>• <b>specify locations and describe spatial relationships using coordinate geometry and other representational systems,</b></li> <li>• <b>apply transformations and use symmetry to analyze mathematical situations, and</b></li> <li>• <b>solve problems using visualization, spatial reasoning, and geometric modeling.</b></li> </ul> |   |
| M.O.6.3.1         | classify lines as parallel, intersecting, perpendicular or skew.  | 1 |
| M.O.6.3.2         | explore the measures of interior angles in polygons and derive the formula to determine the sum.*   | 2 |
| M.O.6.3.3         | bisect a line segment using a compass and straightedge.   | 1 |
| M.O.6.3.4         | draw an angle of a given measure.   | 1 |
| M.O.6.3.5         | create designs using line and rotational symmetry.  | 2 |
| M.O.6.3.6         | define and sketch similar and congruent plane geometric figures.  | 1 |
| M.O.6.3.7         | solve real world problems involving geometric representations.  | 2 |
| <b>Standard 4</b> | <b>Measurement</b>  |   |
| M.S.6.4           | <p>Through communication, representation, reasoning and proof, problem solving, and making connections within and beyond the field of mathematics, students will</p> <ul style="list-style-type: none"> <li>• demonstrate understanding of measurable attributes of objects and the units, systems, and processes</li> </ul>  |   |

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|                   | <p>of measurement, and</p> <ul style="list-style-type: none"> <li>• apply appropriate techniques, tools and formulas to determine measurements.</li> </ul>  |   |
| M.O.6.4.1         | derive approximation for pi using actual measurements.  | 2 |
| M.O.6.4.2         | derive formulas to determine perimeter, circumference and/or area of plane figures including composite figures.   | 2 |
| M.O.6.4.3         | investigate, model and describe surface area of rectangular prisms and cylinders.   | 2 |
| M.O.6.4.4         | solve application problems involving volume of rectangular prisms and cylinders.  | 2 |
| M.O.6.4.5         | construct scale drawings.   | 2 |
| <b>Standard 5</b> | <b>Data Analysis and Probability</b>  |   |
| M.S.6.5           | <p>Through communication, representation, reasoning and proof, problem solving, and making connections within and beyond the field of mathematics, students will</p> <ul style="list-style-type: none"> <li>• formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them,</li> <li>• select and use appropriate statistical methods to analyze data,</li> <li>• develop and evaluate inferences and predictions that are based on models, and</li> <li>• apply and demonstrate an understanding of basic concepts of probability.</li> </ul> |   |
| M.O.6.5.1         | collect, organize, display, read, interpret and analyze data using appropriate graphs and tables.   | 3 |
| M.O.6.5.2         | interpret data and formulate questions using mean, median, mode, and range and determine the effect of outliers on the data.  | 2 |
| M.O.6.5.3         | predict the outcome of a given event using its probability and express that probability as a ratio, decimal or percent.   | 2 |
| M.O.6.5.4         | model combinations and permutations of given situations by creating lists.*   | 2 |

| <b>Grade 7</b> | <b>Mathematics</b>   | <b>DOK Level</b> |
|----------------|--|------------------|
| Standard 1     | Number and Operations  |                  |
| M.S.7.1        | Through communication, representation, reasoning and proof, problem solving, and making connections within and beyond the field of mathematics, students will <ul style="list-style-type: none"> <li>• demonstrate understanding of numbers, ways of representing numbers, and relationships among numbers and number systems,</li> <li>• demonstrate meanings of operations and how they relate to one another, and</li> <li>• compute fluently and make reasonable estimates.</li> </ul> |                  |
| M.O.7.1.1      | compare and order integers, decimals, and fractions using multiple representations (e.g., symbols, manipulatives, graphing on a number line).  | 1                |
| M.O.7.1.2      | evaluate powers, squares, and square roots.  | 1                |
| M.O.7.1.3      | define absolute value and determine its effect on a number or expression.  | 1                |
| M.O.7.1.4      | differentiate between rational and irrational numbers.   | 1                |
| M.O.7.1.5      | apply the commutative, associative, distributive, identity and inverse properties.   | 1                |
| M.O.7.1.6      | analyze and solve application problems with whole numbers, integers, decimals, fractions and percents and justify solutions including using estimation and reasonableness.   | 3                |
| Standard 2     | Algebra  |                  |
| M.S.7.2        | Through communication, representation, reasoning and proof, problem solving, and making connections within and beyond the field of mathematics, students will <ul style="list-style-type: none"> <li>• demonstrate understanding of patterns, relations and functions,</li> <li>• represent and analyze mathematical situations and structures using algebraic symbols,</li> <li>• use mathematical models to represent and understand quantitative relationships, and</li> </ul>          |                  |

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|                   | <ul style="list-style-type: none"> <li>analyze change in various contexts.</li> </ul>   |   |
| M.O.7.2.1         | use inductive reasoning to find missing elements in a variety of arithmetic and geometric patterns including algebraic sequences and series.  | 3 |
| M.O.7.2.2         | simplify and evaluate numerical and algebraic expressions with whole numbers, integers, absolute value and exponents using the order of operations and exponential rules.   | 1 |
| M.O.7.2.3         | add, subtract, multiply and divide monomials with no more than two variables and no exponent greater than two.  | 1 |
| M.O.7.2.4         | justify the rules of working with exponents.  | 2 |
| M.O.7.2.5         | use a spreadsheet to create input/output function tables.   | 2 |
| M.O.7.2.6         | use ratios and proportions to represent and solve application problems.   | 2 |
| M.O.7.2.7         | use and apply scientific notation containing positive and negative exponents.   | 1 |
| M.O.7.2.8         | solve one-step linear equations containing rational numbers with integer solutions.   | 1 |
| M.O.7.2.9         | solve basic inequalities using inverse operations and graph solutions.  | 2 |
| M.O.7.2.10        | plot lines within the Cartesian coordinate plane from a table of values and solve mathematical real world problems.   | 3 |
| M.O.7.2.11        | determine the slope of a line from its graphical representation.  | 1 |
| M.O.7.2.12        | represent algebraically and solve real world application problems and justify solutions.  | 3 |
| <b>Standard 3</b> | <b>Geometry</b>   |   |
| M.S.7.3           | <p><b>Through communication, representation, reasoning and proof, problem solving, and making connections within and beyond the field of mathematics, students will</b></p> <ul style="list-style-type: none"> <li><b>analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships,</b></li> <li><b>specify locations and describe spatial relationships using coordinate geometry and other representational systems,</b></li> <li><b>apply transformations and use symmetry to analyze mathematical situations, and</b></li> <li><b>solve problems using visualization, spatial reasoning, and geometric modeling.</b></li> </ul> |   |
| M.O.7.3.1         | identify and construct angle-pairs (e.g., adjacent, complementary, supplementary, vertical).  | 1 |
| M.O.7.3.2         | identify and construct congruent segments and angles, perpendicular bisectors of segments and angle-  | 2 |

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|                   | bisectors.   |   |
| M.O.7.3.3         | apply and demonstrate line symmetry.   | 1 |
| M.O.7.3.4         | apply transformations (rotations, reflections, translations) to plane figures using graph paper.   | 1 |
| M.O.7.3.5         | develop and solve ratio and proportion problems including scale drawings and similar polygons.   | 2 |
| M.O.7.3.6         | solve mathematical real world problems using compound geometric figures.   | 3 |
| <b>Standard 4</b> | <b>Measurement</b>   |   |
| M.S.7.4           | Through communication, representation, reasoning and proof, problem solving, and making connections within and beyond the field of mathematics, students will <ul style="list-style-type: none"> <li>• demonstrate understanding of measurable attributes of objects and the units, systems, and processes of measurement, and</li> <li>• apply appropriate techniques, tools and formulas to determine measurements.</li> </ul>   |   |
| M.O.7.4.1         | use and apply formulas in problem solving situations involving perimeter, circumference, area, surface area, distance and temperature (Celsius, Fahrenheit).   | 1 |
| M.O.7.4.2         | determine the volume of prisms and cylinders.  | 1 |
| M.O.7.4.3         | use the Pythagorean Theorem to find the length of any side of a right triangle and apply to problem solving situations.  | 2 |
| M.O.7.4.4         | convert units of measurement within and between customary and metric systems.  | 1 |
| M.O.7.4.5         | explore the difference between accuracy and precision in problem solving situations.*  | 1 |
| <b>Standard 5</b> | <b>Data Analysis and Probability</b>   |   |
| M.S.7.5           | Through communication, representation, reasoning and proof, problem solving, and making connections within and beyond the field of mathematics, students will <ul style="list-style-type: none"> <li>• formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them,</li> <li>• select and use appropriate statistical methods to analyze data,</li> <li>• develop and evaluate inferences and predictions that are based on models, and</li> <li>• apply and demonstrate an understanding of basic concepts of probability.</li> </ul> |   |
| M.O.7.5.1         | determine experimental and theoretical probability of an event.  | 2 |

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| M.O.7.5.2 | determine combinations and permutations by constructing sample spaces (e.g., listing, tree diagrams, frequency distribution tables).  | 2 |
| M.O.7.5.3 | collect, organize, graphically represent, and interpret data displays including frequency distributions, line-plots, scatter plots, box and whiskers, and multiple-line graphs. | 2 |

| <b>Grade 8</b>    | <b>Mathematics</b>   | <b>DOK Level</b> |
|-------------------|--|------------------|
| <b>Standard 1</b> | <b>Number and Operations</b>   |                  |
| M.S.8.1           | Through communication, representation, reasoning and proof, problem solving, and making connections within and beyond the field of mathematics, students will <ul style="list-style-type: none"> <li>• demonstrate understanding of numbers, ways of representing numbers, and relationships among numbers and number systems,</li> <li>• demonstrate meanings of operations and how they relate to one another, and</li> <li>• compute fluently and make reasonable estimates.</li> </ul>                                       |                  |
| M.O.8.1.1         | compare and order rational and irrational numbers.   | 1                |
| M.O.8.1.2         | compare and contrast the properties of terminating, repeating, and non-repeating decimals, and conversions between fractions, mixed numbers, and decimals.   | 1                |
| M.O.8.1.3         | use powers, squares, and square roots to solve problems.   | 1                |
| M.O.8.1.4         | analyze and solve application problems with whole numbers, decimals, fractions, percents and integers including, but not limited to, rates, tips, discounts, sales tax and interest and verify solutions using estimation techniques.  | 3                |
| <b>Standard 2</b> | <b>Algebra</b>   |                  |
| M.S.8.2           | Through communication, representation, reasoning and proof, problem solving, and making connections within and beyond the field of mathematics, students will <ul style="list-style-type: none"> <li>• demonstrate understanding of patterns, relations and functions,</li> <li>• represent and analyze mathematical situations and structures using algebraic symbols,</li> <li>• use mathematical models to represent and understand quantitative relationships, and</li> <li>• analyze change in various contexts.</li> </ul> |                  |
| M.O.8.2.1         | solve one and two step linear equations and inequalities with integers, fractions, and decimal solutions.  | 2                |
| M.O.8.2.2         | use ratio and proportion to create and solve equations.  | 2                |
| M.O.8.2.3         | add and subtract polynomials limited to two variables and positive exponents.  | 1                |
| M.O.8.2.4         | generate and apply algebraic equations and expressions to solve application problems using multiple strategies.  | 2                |

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| M.O.8.2.5         | apply inductive and deductive reasoning to write a rule from data in a function table.  | 3 |
| M.O.8.2.6         | graph linear equations and inequalities within the Cartesian coordinate plane using ordered pairs, table of values and appropriate technology.  | 1 |
| M.O.8.2.7         | formulate and apply a rule to generate an arithmetic, geometric and algebraic pattern.  | 2 |
| M.O.8.2.8         | determine the slope of a line given two-points or slope/y-intercept equation ( $y=mx+b$ ).  | 1 |
| M.O.8.2.9         | represent and solve real world problems appropriate for 8 <sup>th</sup> grade using multiple strategies and justify solutions.  | 3 |
| <b>Standard 3</b> | <b>Geometry</b>   |   |
| M.S.8.3           | <p><b>Through communication, representation, reasoning and proof, problem solving, and making connections within and beyond the field of mathematics, students will</b></p> <ul style="list-style-type: none"> <li>• <b>analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships,</b></li> <li>• <b>specify locations and describe spatial relationships using coordinate geometry and other representational systems,</b></li> <li>• <b>apply transformations and use symmetry to analyze mathematical situations, and</b></li> <li>• <b>solve problems using visualization, spatial reasoning, and geometric modeling.</b></li> </ul> |   |
| M.O.8.3.1         | recognize the relationships among corresponding, alternate interior, and alternate exterior angles when parallel lines are cut by a transversal using models, pencil/paper and graphing calculator.   | 1 |
| M.O.8.3.2         | classify polyhedrons according to the number and shape of faces; <b>determine the relationship</b> between vertices, faces and edges.*  | 3 |
| M.O.8.3.3         | identify, apply, and construct perpendicular and angle bisectors.   | 1 |
| M.O.8.3.4         | create geometric patterns including tiling, art design, tessellations and scaling using transformations (rotations, reflections, translations) and predict results of combining, subdividing, and changing shapes of plane figures and solids.  | 3 |
| M.O.8.3.5         | use coordinate geometry to represent <b>and examine</b> properties of similar and congruent figures and graph transformations.  | 2 |

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| M.O.8.3.6         | create scale models using ratio, proportion and similar figures using pencil/paper and dynamic geometry software to determine the proportionality.   | 2 |
| M.O.8.3.7         | describe the intersection of two or more geometric figures in the plane (e.g., intersection of a circle and a line).   | 1 |
| M.O.8.3.8         | describe the cross section of a solid such as a cylinder, cone, and pyramid.   | 1 |
| M.O.8.3.9         | make and test a geometric conjecture about regular polygons.   | 3 |
| <b>Standard 4</b> | <b>Measurement</b>   |   |
| M.S.8.4           | Through communication, representation, reasoning and proof, problem solving, and making connections within and beyond the field of mathematics, students will <ul style="list-style-type: none"> <li>• demonstrate understanding of measurable attributes of objects and the units, systems, and processes of measurement, and</li> <li>• apply appropriate techniques, tools and formulas to determine measurements.</li> </ul>   |   |
| M.O.8.4.1         | <b>develop</b> and apply the formulas for volume of cones, pyramids and spheres.   | 3 |
| M.O.8.4.2         | solve problems involving missing measurements in plane and solid geometric figures using formulas and drawings including irregular figures, models or definitions.   | 2 |
| M.O.8.4.3         | solve right triangle problems using the Pythagorean Theorem and indirect measurement in problem solving situations.  | 3 |
| <b>Standard 5</b> | <b>Data Analysis and Probability</b>   |   |
| M.S.8.5           | Through communication, representation, reasoning and proof, problem solving, and making connections within and beyond the field of mathematics, students will <ul style="list-style-type: none"> <li>• formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them,</li> <li>• select and use appropriate statistical methods to analyze data,</li> <li>• develop and evaluate inferences and predictions that are based on models, and</li> <li>• apply and demonstrate an understanding of basic concepts of probability.</li> </ul> |   |

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| M.O.8.5.1 | use appropriate technology to solve application problems involving combinations and permutations.   | 2 |
| M.O.8.5.2 | investigate interpret the experimental and theoretical probability, including compound probability of a dependent and independent event.  | 2 |
| M.O.8.5.3 | create and extrapolate information from multiple-bar graphs, box and whisker plots, and other data displays using appropriate technology.   | 2 |
| M.O.8.5.4 | analyze problem situations, games of chance, and consumer applications using random and non-random samplings to determine probability, make predictions, and identify sources of bias.    | 3 |
| M.O.8.5.5 | draw inferences, make conjectures and construct convincing arguments, including misuses of statistical or numeric information, based on data analysis of same and different sets of data. | 3 |

| Grade 9-12         | Mathematics: Algebra/Geometry Preparation   | DOK Level |
|--------------------|---|-----------|
| <b>Standard 1</b>  | Number and Operations   |           |
| M.S.AGP.1          | <p>Through communication, representation, reasoning and proof, problem solving, and making connections within and beyond the field of mathematics, students will</p> <ul style="list-style-type: none"> <li>• demonstrate understanding of numbers, ways of representing numbers, and relationships among numbers and number systems,</li> <li>• demonstrate meanings of operations and how they relate to one another, and</li> <li>• compute fluently and make reasonable estimates.</li> </ul>                                       |           |
| M.O.AGP.1.1        | examine and <b>develop fluency</b> with properties of numbers (commutative, associative, distributive, etc).  | 1         |
| M.O.AGP.1.2        | add, subtract, multiply, and divide decimals, integers, fractions and mixed numbers and judge the reasonableness of numerical computations and their results.   | 1         |
| <b>M.O.AGP.1.3</b> | use order relations to compare, order, or locate whole numbers, integers, fractions, and decimals on a number line.   | 1         |
| <b>Standard 2</b>  | Algebra   |           |
| M.S.AGP.2          | <p>Through communication, representation, reasoning and proof, problem solving, and making connections within and beyond the field of mathematics, students will</p> <ul style="list-style-type: none"> <li>• demonstrate understanding of patterns, relations and functions,</li> <li>• represent and analyze mathematical situations and structures using algebraic symbols,</li> <li>• use mathematical models to represent and understand quantitative relationships, and</li> <li>• analyze change in various contexts.</li> </ul> |           |
| M.O.AGP.2.1        | evaluate expressions involving variables, and calculate formulas to solve application problems using function notation.   | 1         |
| M.O..AGP.2.2       | solve equations with at least two operations and confirm the solution.  | 2         |
| <b>Standard 3</b>  | Geometry  |           |
| M.S.AGP.3          | <p><b>Through communication, representation, reasoning and proof, problem solving, and making connections within and beyond the field of mathematics, students will</b></p> <ul style="list-style-type: none"> <li>• <b>analyze characteristics and properties of two- and three-dimensional geometric shapes and</b></li> </ul>  |           |

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|             | <p><b>develop mathematical arguments about geometric relationships,</b></p> <ul style="list-style-type: none"> <li>• <b>specify locations and describe spatial relationships using coordinate geometry and other representational systems,</b></li> <li>• <b>apply transformations and use symmetry to analyze mathematical situations, and</b></li> <li>• <b>solve problems using visualization, spatial reasoning, and geometric modeling.</b></li> </ul> |   |
| M.O.AGP.3.1 | use a compass to construct congruent angles, bisect angles, and bisect line segments.   | 1 |
| M.O.AGP.3.2 | estimate and calculate the circumference and area of a circle.  | 2 |
| M.O.AGP.3.3 | estimate and calculate the area and perimeter of polygons, including those with unknown side lengths.   | 2 |
| M.O.AGP.3.4 | estimate and calculate the surface area and the volume of three-dimensional figures.  | 2 |
| M.O.AGP.3.5 | explore angle relationships: complementary, supplementary, vertical, and adjacent.  | 1 |
| M.O.AGP.3.6 | explore angle relationships involving parallel lines and apply in solving problems (corresponding angles, alternate interior angles, and alternate exterior angles).  | 2 |
| M.O.AGP.3.7 | investigate congruent and similar triangles and apply proportions in problem solving situations.  | 2 |
| M.O.AGP.3.8 | develop and explore circle relationships, emphasizing the vocabulary of circles.?   | 1 |
| Standard 4  | Measurement   |   |
| M.S.AGP.4   | <p>Through communication, representation, reasoning and proof, problem solving, and making connections within and beyond the field of mathematics, students will</p> <ul style="list-style-type: none"> <li>• demonstrate understanding of measurable attributes of objects and the units, systems, and processes of measurement, and</li> <li>• apply appropriate techniques, tools and formulas to determine measurements.</li> </ul>                     |   |
| M.O.AGP.4.1 | estimate, measure, and perform operations involving length, mass, and capacity using customary and metric units.  | 2 |
| M.O.AGP.4.2 | use a protractor to measure and draw angles.  | 1 |
| Standard 5  | Data Analysis and Probability   |   |
| M.S.AGP.5   | Through communication, representation, reasoning and proof, problem solving, and making   |   |

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|             | <p>connections within and beyond the field of mathematics, students will</p> <ul style="list-style-type: none"> <li>• formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them,</li> <li>• select and use appropriate statistical methods to analyze data,</li> <li>• develop and evaluate inferences and predictions that are based on models, and</li> <li>• apply and demonstrate an understanding of basic concepts of probability.</li> </ul> |   |
| M.O.AGP.5.1 | construct and interpret graphs to analyze data and make predictions.  | 2 |
| M.O.AGP.5.2 | use data to determine mean, median, mode, and range.  | 1 |
| M.O.AGP.5.3 | determine the probability of complementary events and mutually exclusive events.  | 2 |

| Grade 9-12 | Mathematics: Algebra I  | DOK Level |
|------------|---|-----------|
| Standard 2 | Algebra   |           |
| M.S.A1.2   | <p>Through communication, representation, reasoning and proof, problem solving, and making connections within and beyond the field of mathematics, students will</p> <ul style="list-style-type: none"> <li>• demonstrate understanding of patterns, relations and functions,</li> <li>• represent and analyze mathematical situations and structures using algebraic symbols,</li> <li>• use mathematical models to represent and understand quantitative relationships, and</li> <li>• analyze change in various contexts.</li> </ul> |           |
| M.O.A1.2.1 | simplify and evaluate algebraic expressions using order of operations and properties of real numbers with justification of steps.   | 1         |
| M.O.A1.2.2 | solve multi-step linear equations in one variable, apply skills toward solving practical problems and judge the reasonableness of solutions.  | 2         |
| M.O.A1.2.3 | solve multi-step linear inequalities in one variable, interpret the results on a number line, apply the skills toward solving practical problems, and judge the reasonableness of solutions.  | 2         |
| M.O.A1.2.4 | solve literal equations for a given variable, apply the skills toward solving practical problems and judge the reasonableness of solutions.   | 2         |

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| M.O.A1.2.5  | analyze a given set of data for the existence of a pattern numerically, algebraically and graphically;<br>determine the domain and range; and determine if the relation is a function using function notation when appropriate. | 2 |
| M.O.A1.2.6  | solve absolute value equations in one variable and interpret the results on a number line.  | 2 |
| M.O.A1.2.7  | <b>investigate</b> the laws of exponents and perform operations on expressions with integral exponents.   | 1 |
| M.O.A1.2.8  | determine the slope of a line given an equation of a line, the graph of a line and two points.  | 1 |
| M.O.A1.2.9  | graph linear equations using slope-intercept, point slope, and x- and y-intercepts.   | 1 |
| M.O.A1.2.10 | determine the equation of a line given a graph of a line, two points on the line, the slope and a point, and the slope and y-intercept.   | 1 |
| M.O.A1.2.11 | solve systems of linear equations graphically and numerically using the elimination method and the substitution method.   | 2 |
| M.O.A1.2.12 | add and subtract polynomials.   | 1 |
| M.O.A1.2.13 | multiply and divide binomials by binomials or monomials.  | 1 |

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| M.O.A1.2.14 | use area models and graphical representations to develop and explain appropriate methods of factoring.   | 2 |
| M.O.A1.2.15 | simplify square roots into both exact and approximate forms.   | 1 |
| M.O.A1.2.16 | solve quadratic equations by graphing, factoring and quadratic formula and draw reasonable conclusions about a situation being modeled.  | 2 |
| M.O.A1.2.17 | <b>examine</b> situations that can be modeled by a rational expression; add, subtract, multiply and divide simple rational expressions and determine when an expression is undefined.  | 1 |
| M.O.A1.2.18 | collect, organize, interpret data and predict outcomes using the mean, mode, median, and range including problems with missing data values.  | 2 |
| M.O.A1.2.19 | perform a linear regression, identify the equation for the line of regression, examine the correlation coefficient to determine how well the line fits the data and use the equation to predict specific values of a variable. | 2 |
| M.O.A1.2.20 | compute and interpret the expected value of random variables in simple cases using simulations and rules of probability.   | 2 |

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| M.O.A1.2.21 | understand histograms, box plots, scatter plots and normal distribution curves and use them to display and interpret data.   | 2 |
| M.O.A1.2.22 | compute a probability when the event and/or sample space are not given or obvious.   | 2 |
| M.O.A1.2.23 | <b>use multiple representations, such as words, graphs, tables of values and equations, to solve practical problems; explain how the representations are related to each other as well as to the problems.</b> | 2 |

| Grade 9-12         | Mathematics: Applied Mathematics I  | DOK Level |
|--------------------|---|-----------|
| Standard 2         | Algebra   |           |
| M.S.AM1.2          | <p>Through communication, representation, reasoning and proof, problem solving, and making connections within and beyond the field of mathematics, students will</p> <ul style="list-style-type: none"> <li>• demonstrate understanding of patterns, relations and functions,</li> <li>• represent and analyze mathematical situations and structures using algebraic symbols,</li> <li>• use mathematical models to represent and understand quantitative relationships, and</li> <li>• analyze change in various contexts.</li> </ul> |           |
| M.O.AM1.2.1        | solve practical problems involving computation using estimation.  | 2         |
| M.O.AM1.2.2        | write numbers in scientific notation and combine numbers written in scientific notation to solve practical problems.  | 1         |
| M.O.AM1.2.3        | <b>distinguish</b> between counting and measuring using precision measurement tools.  | 1         |
| M.O.AM1.2.4        | solve practical problems and interpret results using rational numbers and vectors.  | 2         |
| M.O.AM1.2.5        | simplify and evaluate algebraic expressions using order of operations and properties of real numbers with justification of steps.   | 1         |
| M.O.AM1.2.6        | translate word phrases into algebraic expressions or word sentences into equations and inequalities.  | 1         |
| <b>M.O.AM1.2.7</b> | <b>justify steps in the solving of equations based on the properties of real numbers.</b>   | 2         |
| M.O.AM1.2.8        | solve <b>literal</b> equations for a given variable and apply the skills toward solving practical problems and judge the reasonableness of solutions.   | 2         |
| M.O.AM1.2.9        | solve practical problems using a four-step problem solving approach.  | 2         |
| M.O.AM1.2.10       | solve multi-step linear equations in one variable, apply skills toward solving practical problems, and  | 2         |

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|                   | judge the reasonableness of solutions.   |   |
| M.O.AM1.2.11      | solve multi-step linear inequalities in one variable, interpret the results on a number line, <b>apply the skills toward</b> solving practical problems and judge the reasonableness of solutions.   | 2 |
| M.O.AM1.2.12      | solve absolute value equations in one variable and interpret the results on a number line.   | 2 |
| M.O.AM1.2.13      | collect, organize, interpret data and predict outcomes using the mean, mode and range including problems with missing data values.   | 3 |
| M.O.AM1.2.14      | simplify square roots into both exact and approximate forms.   | 1 |
| M.O.AM1.2.15      | investigate the laws of exponents and perform operations on expressions with integral exponents.   | 1 |
| M.O.AM1.2.16      | compute and interpret the expected value of random variables in simple cases using simulations and rules of probability.   | 2 |
| M.O.AM1.2.17      | compute a probability when the event and/or sample space are not given or obvious.   | 2 |
| <b>Standard 2</b> | Algebra  |   |
| M.S.AM2.2         | Through communication, representation, reasoning and proof, problem solving, and making connections within and beyond the field of mathematics, students will <ul style="list-style-type: none"> <li>• demonstrate understanding of patterns, relations and functions,</li> <li>• represent and analyze mathematical situations and structures using algebraic symbols,</li> <li>• use mathematical models to represent and understand quantitative relationships, and</li> <li>• analyze change in various contexts.</li> </ul> |   |
| M.O.AM2.2.1       | analyze a given set of data for the existence of a pattern numerically, algebraically and graphically; determine the domain and range; and determine if the relation is a function using function notation when appropriate.   | 2 |

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| M.O.AM2.2.2        | determine the slope of a line given an equation of a line, the graph of a line and two points.   | 1 |
| M.O.AM2.2.3        | graph linear equations using slope-intercept, point slope and x- and y-intercepts.   | 1 |
| M.O.AM2.2.4        | write an equation of a line given graph of a line, two points on the line, the slope and a point and the slope and y-intercept.  | 2 |
| <b>M.O.AM2.2.5</b> | <b><i>solve systems of linear equations graphically and numerically using the elimination method and the substitution method.</i></b>  | 2 |
| M.O.AM2.2.6        | add and subtract polynomials.  | 1 |
| M.O.AM2.2.7        | multiply and divide binomials by binomials or monomials.   | 1 |
| M.O.AM2.2.8        | use area models and graphical representations to develop and explain appropriate methods of factoring.   | 2 |
| M.O.AM2.2.9        | solve quadratic equations by graphing, factoring and the quadratic formula and draw reasonable conclusions about a situation being modeled.  | 3 |
| M.O.AM2.2.10       | examine situations that can be modeled by a rational expression, add, subtract, multiply and divide simple rational expressions and determine when an expression is undefined.   | 1 |
| M.O.AM2.2.11       | understand box plots, histograms, scatter plots and use them to display and interpret data.  | 2 |
| M.O.AM2.2.12       | perform a linear regression, identify the equation for the line of regression, examine the correlation coefficient to determine how well the line fits the data and use the equation to predict specific values of a variable. | 2 |

| Grade 9-12       | Mathematics: Geometry and Applied Geometry  | DOK Level |
|------------------|---|-----------|
| Standard 3       | Geometry  |           |
| M.S.G.3          | <p><b>Through communication, representation, reasoning and proof, problem solving, and making connections within and beyond the field of mathematics, students will</b></p> <ul style="list-style-type: none"> <li>• <b>analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships,</b></li> <li>• <b>specify locations and describe spatial relationships using coordinate geometry and other representational systems,</b></li> <li>• <b>apply transformations and use symmetry to analyze mathematical situations, and</b></li> <li>• <b>solve problems using visualization, spatial reasoning, and geometric modeling.</b></li> </ul> |           |
| M.O.G.3.1        | represent geometric figures, such as points, lines, planes, segments, rays, and angles pictorially with proper identification and distinguish between undefined and defined terms.  | 1         |
| M.O.G.3.2        | differentiate between and apply inductive and deductive reasoning.  | 3         |
| <b>M.O.G.3.3</b> | <b>use the basic concepts of symbolic logic including identifying the converse, inverse, and contrapositive of a conditional statement and test the validity of conclusions with methods that include Venn Diagrams.</b>  | 3         |
| <b>M.O.G.3.4</b> | <b>construct logical arguments using both formal and informal methods with direct and indirect reasoning.</b>   | 3         |
| M.O.G.3.5        | apply definitions, theorems, and postulates related to such topics as angles formed by perpendicular lines, and complementary, supplementary, and vertical angles to construct formal and informal proofs.  | 3         |
| M.O.G.3.6        | compare and contrast the relationships between angles formed by two lines cut by a transversal when lines are and are not parallel, and use the results to develop methods to show parallelism.   | 2         |

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| M.O.G.3.7  | investigate and justify congruence relationships with an emphasis on triangles.  | 2 |
| M.O.G.3.8  | explore and identify general properties of convex and concave quadrilaterals; develop, compare and contrast the properties of parallelograms, rectangles, rhombuses, squares, and trapezoids.  | 2 |
| M.O.G.3.9  | using similar figures, apply proportions in problem solving situations that include two and three dimensions.  | 2 |
| M.O.G.3.10 | investigate measures of angles and lengths of segments to determine the existence of a triangle (triangle inequality) and establish the relationship between the order of sides and angles.  | 2 |
| M.O.G.3.11 | derive the basis for the trigonometric ratios by applying properties of similar triangles with justification and using the results to construct scale drawings and find unknown side lengths, angle measures and inaccessible heights and distances. | 3 |
| M.O.G.3.12 | apply the Pythagorean Theorem and its converse in solving practical problems and deriving the special right triangle relationships.  | 2 |
| M.O.G.3.13 | investigate measures of angles formed by chords, tangents, and secants of a circle and the relationship to its arcs.?  | 2 |
| M.O.G.3.14 | given a polygon, find angle measures of interior and exterior angles; find the length of sides from given data; and use properties of regular polygons to find missing data.   | 2 |

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| M.O.G.3.15        | develop properties of tessellating figures and use those properties to tessellate the plane.  | 2 |
| M.O.G.3.16        | use nets and other manipulatives to <b>develop</b> and apply formulas for area, perimeter, surface area, and volume and apply them in the modeling of practical problems.   | 2 |
| <b>M.O.G.3.17</b> | <b>develop and apply concepts of analytical geometry such as formulas for distance, slope, and midpoint and apply these to finding dimensions of polygons on the coordinate plane.</b>  | 2 |
| M.O.G.3.18        | using various methods, construct a triangle's medians, altitudes, angle and perpendicular bisectors; identify conjectures and develop mathematical arguments about their relationships.   | 2 |
| M.O.G.3.19        | using transformational geometry, create a reflection, translation, rotation, glide reflection and dilation of a figure; and apply transformations and use symmetry to <b>analyze</b> mathematical situations such as congruency and similarity. | 3 |
| M.O.G.3.20        | compare and contrast Euclidean geometry to other geometries such as spherical.  | 2 |
| M.O.G.3.21        | find or approximate the area of irregularly shaped regions.   | 2 |
| M.O.G.3.22        | using the Cartesian coordinate system, find the dimensions of a polygon given the coordinates of the polygon.   | 2 |

| Grade 9-12        | Mathematics: Algebra II   | DOK Level |
|-------------------|---|-----------|
| Standard 2        | Algebra   |           |
| M.S.A2.2          | <p>Through communication, representation, reasoning and proof, problem solving, and making connections within and beyond the field of mathematics, students will</p> <ul style="list-style-type: none"> <li>• demonstrate understanding of patterns, relations and functions,</li> <li>• represent and analyze mathematical situations and structures using algebraic symbols,</li> <li>• use mathematical models to represent and understand quantitative relationships, and</li> <li>• analyze change in various contexts.</li> </ul> |           |
| M.O.A2.2.1        | determine equations of lines including parallel, perpendicular, vertical and horizontal lines.  | 1         |
| M.O.A2.2.2        | factor higher order polynomials by applying various methods, including factoring by grouping, the sum and difference of two cubes and graphical representation.   | 1         |
| M.O.A2.2.3        | develop the complex number system and simplify powers of 'i'.   | 3         |
| M.O.A2.2.4        | perform basic operations with complex numbers and give answers as complex numbers in simplest form.   | 1         |
| <b>M.O.A2.2.5</b> | <b>simplify radicals and expressions involving fractional exponents and convert between the two forms.</b>  | 1         |
| M.O.A2.2.6        | solve quadratic equations over the set of complex numbers: apply the techniques of factoring, completing the square, and the quadratic formula; use the discriminate to determine the number and nature of the roots; confirm the solutions numerically and graphically; and apply to practical problems.   | 2         |
| M.O.A2.2.7        | develop and use the appropriate field properties of matrices by adding, subtracting, and multiplying; solve a system of linear equations using matrices; and apply skills toward solving practical problems.  | 2         |
| <b>M.O.A2.2.8</b> | <b>solve equations containing radicals and exponents.</b>   | 1         |
| M.O.A2.2.9        | define a function and find its zeros; express the domain and range using interval notation; find the inverse of a function; find the value of a function for a given element in its domain; and perform basic operations on functions including composition of functions.   | 2         |
| M.O.A2.2.10       | explore families of functions and their transformations; recognize linear, quadratic, radical, absolute value, step, piece-wise and exponential functions; convert among words, graphs, tables and equations in practical problems.   | 2         |

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| M.O.A2.2.11        | solve quadratic inequalities, graph their solution sets and express this solution using interval notation.   | 2 |
| M.O.A2.2.12        | solve and graph the solution set of systems of linear inequalities in two variables by finding the maximum or minimum values of a function over the feasible region using linear programming techniques. | 2 |
| <b>M.O.A2.2.13</b> | <b>solve practical problems involving direct, inverse and joint variation.</b>   | 2 |
| <b>M.O.A2.2.14</b> | <b>explore the conic sections; recognize, identify and sketch the graphs of a parabola, circle, ellipse and hyperbola and convert between graphs and equations.</b>                                      | 2 |
| <b>M.O.A2.2.15</b> | <b>solve absolute value inequalities graphically, numerically and algebraically and express the solution set in interval notation.</b>   | 2 |
| M.O.A2.2.16        | define a logarithmic function, transform between exponential and logarithmic forms and apply the basic properties of logarithms to simplify or expand an expression.                                     | 2 |
| M.O.A2.2.17        | perform a quadratic regression, determine the regression equation and use the results to predict specific values of a variable.  | 3 |
| M.O.A2.2.18        | develop recursive models for representing and investigating patterns and sequences.  | 2 |

| Grade 9-12 | Mathematics: Conceptual Mathematics   | DOK Level |
|------------|---|-----------|
| Standard 2 | Algebra   |           |
| M.S.CM.2   | <p>Through communication, representation, reasoning and proof, problem solving, and making connections within and beyond the field of mathematics, students will</p> <ul style="list-style-type: none"> <li>• demonstrate understanding of patterns, relations and functions,</li> <li>• represent and analyze mathematical situations and structures using algebraic symbols,</li> <li>• use mathematical models to represent and understand quantitative relationships, and</li> <li>• analyze change in various contexts.</li> </ul>   |           |
| M.O.CM.2.1 | develop a variety of problem solving strategies (e.g., draw a diagram, look for a pattern, work backwards).   | 2         |
| M.O.CM.2.2 | interpret graphs of functions including linear, quadratic, and exponential.   | 2         |
| M.O.CM.2.3 | solve application problems using linear, quadratic and exponential functions with emphasis on data collection and analysis.   | 3         |
| M.O.CM.2.4 | use appropriate formulas to solve workplace problems.   | 2         |
| M.O.CM.2.5 | calculate costs, simple and compound interest, finance charges, loan payments and taxes.  | 2         |
| M.O.CM.2.6 | compare various methods of investing money.   | 2         |
| Standard 3 | Geometry  |           |
| M.S.CM.3   | <p><b>Through communication, representation, reasoning and proof, problem solving, and making connections within and beyond the field of mathematics, students will</b></p> <ul style="list-style-type: none"> <li>• <b>analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships,</b></li> <li>• <b>specify locations and describe spatial relationships using coordinate geometry and other representational systems,</b></li> <li>• <b>apply transformations and use symmetry to analyze mathematical situations, and</b></li> <li>• <b>solve problems using visualization, spatial reasoning, and geometric modeling.</b></li> </ul> |           |

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| M.O.CM.3.1        | apply concepts of geometry including the Pythagorean Theorem, similar triangles, and right triangle trigonometry.  | 2 |
| M.O.CM.3.2        | solve workplace problems involving perimeter, area, surface area and volume.   | 2 |
| M.O.CM.3.3        | investigate the applications of various geometric shapes and patterns to art, architecture, and nature. ?  | 2 |
| <b>Standard 3</b> | Data Analysis and Probability  |   |
| M.S.CM.5          | Through communication, representation, reasoning and proof, problem solving, and making connections within and beyond the field of mathematics, students will <ul style="list-style-type: none"> <li>• formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them,</li> <li>• select and use appropriate statistical methods to analyze data,</li> <li>• develop and evaluate inferences and predictions that are based on models, and</li> <li>• apply and demonstrate an understanding of basic concepts of probability.</li> </ul> |   |
| M.O.CM.5.1        | relate mathematical content to its historical development.   | 2 |
| M.O.CM.5.2        | integrate other disciplines into the study of mathematics through simulations, research, and projects.   | 3 |
| <b>M.O.CM.5.3</b> | <b>determine possible outcomes using tree diagrams and the counting principles of permutations and combinations.</b>   | 2 |
| M.O.CM.5.4        | apply the basic probability rules in expressing the chances of events occurring using technology when appropriate.   | 2 |
| M.O.CM.5.5        | create and interpret data using various methods of displaying numerical data, including frequency distributions, graphs, histograms, stem-and-leaf plots, and box-and-whiskers plots, using technology when appropriate.   | 2 |
| M.O.CM.5.6        | relate the measures of central tendency and the measures of dispersion to a normal distribution.   | 2 |
| M.O.CM.5.7        | apply the measures of central tendency and the measures of dispersion to workplace situations.   | 2 |
| M.O.CM.5.8        | use statistical tools for workplace applications such as quality control, marketing and predicting trends.   | 3 |

| Grade 9-12       | Mathematics: Trigonometry   | DOK Level |
|------------------|---|-----------|
| Standard 3       | Geometry  |           |
| M.S.T.3          | <p><b>Through communication, representation, reasoning and proof, problem solving, and making connections within and beyond the field of mathematics, students will</b></p> <ul style="list-style-type: none"> <li>• <b>analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships,</b></li> <li>• <b>specify locations and describe spatial relationships using coordinate geometry and other representational systems,</b></li> <li>• <b>apply transformations and use symmetry to analyze mathematical situations, and</b></li> <li>• <b>solve problems using visualization, spatial reasoning, and geometric modeling.</b></li> </ul> |           |
| M.O.T.3.1        | apply and connect the right triangle definition of the six trigonometric functions of an angle to the circular functions and determine the values of the functions of an angle in standard position given a point on the terminal side of the angle.  | 2         |
| M.O.T.3.2        | determine the values of the other trigonometric functions, given the value of one trigonometric function.   | 1         |
| <b>M.O.T.3.3</b> | <b>develop fluent recall of the values of the six trigonometric functions of special angles as related to the unit circle.</b>  | 1         |
| M.O.T.3.4        | use a calculator to determine the values of the trigonometric functions for any angle and to determine the measure of an angle given the value of one of its trigonometric functions.   | 1         |
| M.O.T.3.5        | convert angle measures from radians to degrees and vice versa.  | 1         |
| M.O.T.3.6        | verify trigonometric identities using various methods including basic identities and graphical representation.  | 3         |
| M.O.T.3.7        | solve trigonometric equations that include both infinite solutions and solutions with a restricted domain.  | 3         |
| M.O.T.3.8        | find the value of inverse trigonometric functions.  | 2         |
| M.O.T.3.9        | find the area of a triangle given the measures of two sides and the included angle or the measures of three sides (Heron's formula).  | 1         |

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| M.O.T.3.10 | express complex numbers in polar form: perform operations including adding, subtracting, multiplying, and dividing; evaluate powers and roots of complex numbers using De Moivre's Theorem; and graph complex numbers.  | 2 |
| M.O.T.3.11 | solve practical problems involving triangles using the trigonometric functions, the Law of Sines, and the Law of Cosines.   | 2 |
| M.O.T.3.12 | graph and recognize the graph of the six trigonometric functions. Given an equation in the form of $y=a\sin(bx+c)+d$ , identify the domain and range; determine the period, phase shift, amplitude and vertical shift; and sketch at least one period of the graph. | 2 |
| M.O.T.3.13 | model periodic data sets using graphs, tables, and equations.   | 2 |
| M.O.T.3.14 | recognize and graph the inverse of trigonometric functions. Restrictions on the domain will be included.  | 2 |
| M.O.T.3.15 | <b>develop</b> and use formulas such as sum or difference of two angles, double-angle, and half-angle.  | 2 |
| M.O.T.3.16 | <b>perform mathematical operations with vectors and apply vectors to solve practical problems.</b>  | 2 |

| Grade 9-12  | Mathematics: Probability and Statistics   | DOK Level |
|-------------|---|-----------|
| Standard 5  | Data Analysis and Probability   |           |
| M.S.PS.5    | <p>Through communication, representation, reasoning and proof, problem solving, and making connections within and beyond the field of mathematics, students will</p> <ul style="list-style-type: none"> <li>• formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them,</li> <li>• select and use appropriate statistical methods to analyze data,</li> <li>• develop and evaluate inferences and predictions that are based on models, and</li> <li>• apply and demonstrate an understanding of basic concepts of probability.</li> </ul> |           |
| M.O.PS.5.1  | distinguish between experimental and theoretical probability.   | 2         |
| M.O.PS.5.2  | create and interpret data using various methods of displaying circle graphs, histograms, and frequency curves, and make predictions about outliers.   | 3         |
| M.O.PS.5.3  | determine possible outcomes using tree diagrams and the counting principles of permutations and combinations.   | 2         |
| M.O.PS.5.4  | express the chances of events occurring either in terms of a probability or odds.   | 2         |
| M.O.PS.5.5  | use the normal distribution and the binomial distribution including Pascal's triangle, to determine probability of events.  | 2         |
| M.O.PS.5.6  | interpret and calculate measures of central tendency (mean, median, and mode) from data presented in a variety of forms such as charts, tables, and graphs or from data created through experimentation.  | 2         |
| M.O.PS.5.7  | interpret and calculate measures of dispersions (range and standard deviation) from data presented in a variety of forms such as charts, tables and graphs or from data created through experimentation.  | 2         |
| M.O.PS.5.8  | describe individual performances in terms of percentiles, z-scores, and t- scores.  | 2         |
| M.O.PS.5.9  | describe the role of sampling, randomness, bias, and sample size in data collection and interpretation.   | 2         |
| M.O.PS.5.10 | test the validity of a hypothesis using statistical concepts including a t-test.  | 3         |
| M.O.PS.5.11 | determine the correlation values for given data or for data generated by students and use the results to describe the association of the variables within the given data. Identify whether this association is systematic or predictable.   | 2         |
| M.O.PS.5.12 | calculate the Chi-Square values for a given population.   | 2         |

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| M.O.PS.5.13 | perform a regression analysis on a set of data, either given or created through experimentation, and use the results to predict specific values of a variable Identify the regression equation. | 3 |
| M.O.PS.5.14 | perform an analysis of variance (ANOVA) and interpret the results.  | 3 |

| Grade 9-12        | Mathematics: Pre-calculus   | DOK Level |
|-------------------|---|-----------|
| <b>Standard 2</b> | Algebra   |           |
| M.S.PC.2          | <p>Through communication, representation, reasoning and proof, problem solving, and making connections within and beyond the field of mathematics, students will</p> <ul style="list-style-type: none"> <li>• demonstrate understanding of patterns, relations and functions,</li> <li>• represent and analyze mathematical situations and structures using algebraic symbols,</li> <li>• use mathematical models to represent and understand quantitative relationships, and</li> <li>• analyze change in various contexts.</li> </ul> |           |
| M.O.PC.2.1        | investigate and sketch the graphs of polynomials and rational functions using the characteristics of zeros, upper and lower bounds, y-intercepts, symmetry, asymptotes and end behavior, maximum and minimum points and domain and range.   | 2         |
| M.O.PC.2.2        | solve higher order polynomial equations utilizing techniques such as Descartes' Rule of Signs, upper and lower bounds and the Rational Root Theorem.  | 2         |
| M.O.PC.2.3        | expand binomials with positive integral exponents by the use of Pascal's triangle and the Binomial Theorem.   | 1         |
| M.O.PC.2.4        | establish the inverse relationship between exponential and logarithmic functions; graph related functions and include their domain and range using interval notation.   | 2         |
| M.O.PC.2.5        | compare laws of exponents to properties of logarithms; solve equations and practical problems involving exponential and logarithmic expressions, including natural and common logarithms; confirm solutions graphically and numerically.  | 2         |
| M.O.PC.2.6        | solve problems involving the sum of finite and infinite sequences and series, including Sigma notation.   | 3         |
| M.O.PC.2.7        | use tables of values, graphs, conjectures, algebraic methods and numerical substitution to find or estimate the limit of a function, a sequence or a series.  | 2         |

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| M.O.PC.2.8        | examine the geometry of vectors, perform mathematical operations with vectors and use vectors to solve practical problems.  | 3 |
| M.O.PC.2.9        | <b>apply the method of mathematical induction to prove formulas and statements.</b>   | 3 |
| M.O.PC.2.10       | <b>apply parametric methods to represent motion of objects.</b>   | 2 |
| M.O.PC.2.11       | use multiple representations, such as words, graphs, tables and equations, to solve practical, logarithmic, exponential, polynomial, rational, radical, problems; explain how the representations are related to each other, as well as to the problem.   | 3 |
| <b>Standard 3</b> | <b>Geometry</b>   |   |
| M.S.PC.3          | <p><b>Through communication, representation, reasoning and proof, problem solving, and making connections within and beyond the field of mathematics, students will</b></p> <ul style="list-style-type: none"> <li>• analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships,</li> <li>• specify locations and describe spatial relationships using coordinate geometry and other representational systems,</li> <li>• apply transformations and use symmetry to analyze mathematical situations, and</li> <li>• solve problems using visualization, spatial reasoning, and geometric modeling.</li> </ul> |   |
| M.O.PC.3.1        | <b>graph functions and conic sections using transformations.</b>  | 2 |
| M.O.PC.3.2        | <b>investigate properties and solve practical problems of the conic sections.</b>   | 2 |
| <b>Standard 5</b> | <b>Data Analysis and Probability</b>  |   |
| M.S.PC.5          | <p>Through communication, representation, reasoning and proof, problem solving, and making connections within and beyond the field of mathematics, students will</p> <ul style="list-style-type: none"> <li>• formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them,</li> <li>• select and use appropriate statistical methods to analyze data,</li> <li>• develop and evaluate inferences and predictions that are based on models, and</li> <li>• apply and demonstrate an understanding of basic concepts of probability.</li> </ul>   |   |

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| M.O.PC.5.1 | perform an exponential and or logarithmic regression analysis on a set of data, write the regression equation and use the results to predict specific values of a variable. | 3 |
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